Emilio Huggins 11/22/2025 Active Rocket Controls Motor Centering Ring Stress/Strain Analysis Section 001

1. Introduction

Our capstone project is a Level 1 rocket that uses actively controlled fin canards and several 3D-printed structural components. During boost, the solid rocket motor pushes against the internal structure of the airframe through a motor centering ring. If this ring were to crush or crack, the motor could shift relative to the airframe, leading to loss of stability or structural failure.

This memo presents an analytical evaluation of the motor centering ring for our rocket. The goal is to verify that the current geometry safely transfers the thrust of our H 100 solid rocket motor to the airframe and to compare three materials: 3D-printed PLA filament, 6061-T6 aluminum, and fiberglass. The ring is modeled as a loaded annulus subject to axial compression from the motor's thrust. For each material, the average compressive stress, corresponding strain, and factor of safety with respect to yielding are calculated. The results are then used to justify which material is most appropriate for flight and to demonstrate that the design is structurally viable.

A detailed CAD model and drawing of the centering ring, including all dimensions, is provided in Figure 1.

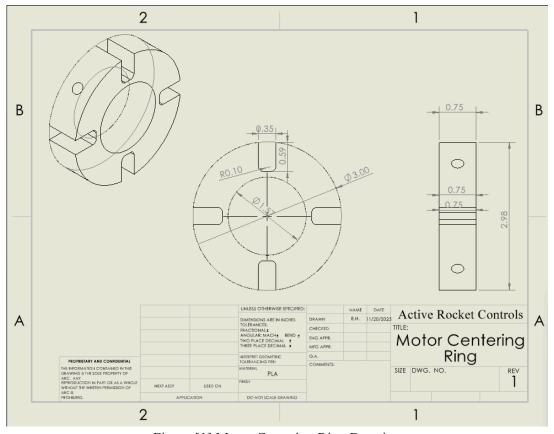


Figure [1] Motor Centering Ring Drawing

2. Geometry and Loading Description

The centering ring is a circular plate with a center bore for the motor mount tube and four radial slots that border the surrounding structure (Figure 1). The dimensions from the drawing (all in inches) are:

Outer diameter: D₀ = 3.00 in
 Inner diameter: D_i = 1.57 in

For this analysis, a conservative design load of

$$F = 613 N \approx 138 lbf$$

is used, designated as the maximum thrust force. The motor's forward closure is assumed to press uniformly against the annular face of the centering ring between the inner and outer diameters. The axial load path is from the motor to the centering ring, and from the ring into the rocket body tube through adhesive or mechanical fasteners.

3. Assumptions and Modeling Approach

To make the problem tractable and clearly connect it to coursework in mechanics of materials and design, the following assumptions are used:

- 1) **Static peak-load condition.** Only the maximum thrust F = 613 N is considered. Dynamic effects (vibration, thrust oscillations, landing loads) are neglected. This produces a conservative stress estimate focused on motor thrust.
- 2) Uniform pressure over the annular face. The thrust is taken as uniformly distributed over the annular area between D_i and D_o . Local stress concentrations near the four slots are neglected in the primary calculation but are discussed qualitatively in Section 7.
- 3) **Slots and small holes are neglected in the area.** The slots and fastener holes remove a small portion of the annular area. Neglecting them slightly underestimates the stress, but the reduction in area is small relative to the overall ring.
- 4) **Linear-elastic material behavior.** PLA, aluminum, and fiberglass are modeled as homogeneous and linearly elastic in the loading direction with a single Young's modulus E. For the fiberglass, properties are taken in the fiber-dominated plane of the ring.

- 5) **Yield-based failure criterion.** The factor of safety is defined as the ratio of material yield (or design) strength to the computed average stress. Typical room-temperature values from material datasheets are used
- 6) **Perfect bonding to the airframe.** The adhesive or mechanical attachment between the centering ring and airframe is assumed stronger than the ring itself. Only the ring material is evaluated in this memo.

4. Variable Definitions

| Symbol | Definition Unit | | |
|------------|--|-----------------|--|
| F | Maximum motor thrust (design load) | | |
| D_{0} | Outer diameter of centering ring | in | |
| D_{i} | Inner diameter (motor-tube bore) in | | |
| A | Annular load-bearing area | in ² | |
| σ | Average compressive stress in ring | psi | |
| ε | Axial strain | - | |
| E | Young's modulus of material | psi | |
| σ_y | Yield (or design) strength of material | psi | |
| N | Factor of safety with respect to yield | <u>-</u> | |

Table [1] Variable Definitions

5. Material Properties

Properties for each material are summarized below. Values fall within ranges reported in literature and datasheets.

- PLA (3D-printed filament): typical tensile strength $\sigma_y = 65 \, MPa$ and modulus $E \approx 3 \, GPa$ which corresponds to 9420 *psi* and 4.35 * 10⁵ *psi* respectively.
- 6061-T6 aluminum: yield strength, elastic modulus $E \approx 69 \, GPa \approx 1.0 * 10^7 \, psi$
- **Fiberglass:** tensile strength is taken as 30 ksi and E as 16 GPa $\approx 2.32 * 10^7 psi$

6. Governing Equations

6.1 Annular Load-Bearing Area

Assuming the motor thrust is transmitted over the annular face between D_i and D_o , the load-bearing area is

(1)

$$A = \frac{\pi}{4} (D_0^2 - D_i^2)$$

6.2 Average Compressive Stress

The average compressive stress in the centering ring is then

(2)

$$\sigma = \frac{F}{A}$$

6.3 Axial Strain

Assuming linear elasticity, the axial strain is

(3)

$$\varepsilon = \frac{\sigma}{E}$$

6.4 Factor of Safety

The factor of safety with respect to yielding is computed as

(4)

$$N = \frac{\sigma_y}{\sigma}$$

Equations (1)–(4) are standard relations from mechanics of materials and allow an independent reader to verify every step of the analysis.

7. Sample Calculation

A full sample calculation is provided here for the PLA centering ring using the conservative design load $F = 138 \, lbf$.

Step 1 – Annular area

$$A = \frac{\pi}{4} (D_0^2 - D_i^2)$$
$$= \frac{\pi}{4} (3.00^2 - 1.57^2) in^2 \approx 5.13 in^2$$

Step 2 – Compressive stress

$$\sigma = \frac{F}{A}$$
= $\frac{138 \, lbf}{5.13 \, in^2} = 26.9 \, psi$

Step 3 - Axial Strain in PLA

$$\varepsilon_{PLA} = \frac{\sigma}{E}$$

$$= \frac{26.9 \, psi}{4.35 \times 10^5 \, psi} = 6.2 \times 10^{-5}$$

Step 4 – FOS for PLA

Using
$$\sigma_v = 7250 \, psi$$
:

$$N_{PLA} = \frac{\frac{\sigma_{y}}{\sigma}}{\sigma}$$

$$= \frac{7250 \, psi}{26.9 \, psi} = 2.7 \, * \, 10^{2}$$

So even the weakest of the three materials has a factor of safety of roughly 270 under the 613 N load.

The same process is repeated for 6061-T6 aluminum and fiberglass- the values summarized in Table 2.

Sensitivity check - Reduced effective area

The annular area computation assumes that the entire ring face shares the load equally. In reality, the four inner webs between the radial slots may carry a larger portion of the thrust. To bound this effect, a second case was examined where the effective area is reduced to 0.69 in², approximately one-eighth of the full annulus. In that case,

$$\sigma_{reduced} = \frac{138}{0.69} \approx 200 \, psi$$

The resulting factors of safety are still large:

- PLA: N≈36
- Aluminum: N≈200
- Fiberglass: N≈355

This sensitivity check shows that even with a very conservative assumption about the contact area, the ring remains safe in all three materials.

8. Data Presentation

Using Equations (1)–(4) and the properties in Section 5, the stresses, strains, and factors of safety for the full-annulus model are summarized in Table 1.

Table 1 – Calculated stress, strain, and factor of safety for centering ring (F=613 N \approx 138 lbf)

| Material | E (PSI) | σ (PSI) | σ_y (PSI) | 3 | N |
|---------------------|------------------------|---------|------------------|------------------------|------|
| PLA (3D Printed) | 4.35 * 10 ⁵ | 7250 | 26. 9 | 6.2 * 10 ⁻⁵ | 270 |
| 6060-T6 Aluminum | 1.00 * 10 ⁷ | 40000 | 26. 9 | 2.7 * 10 ⁻⁶ | 1490 |
| Fiberglass | 1.04 * 10 | 71000 | 26.9 | 2.6 * 10 -6 | 2640 |

Table [2] Data Presentation

9. Results and Discussion

The analysis shows that the average compressive stress in the motor centering ring under the conservative design load of 613 N is approximately 27 psi when the thrust is assumed to be distributed over the full annular face. This stress is extremely small relative to the yield strengths of the three materials considered. Even when the effective area is artificially reduced to 0.69 in², resulting in a stress of 200 psi, the factors of safety remain well above 30 for PLA and greater than 200 for aluminum and fiberglass.

The corresponding elastic strains are also very small—on the order of 10^{-5} to 10^{-6} . This indicates that the ring will not noticeably compress under load, and the motor will remain well aligned with the airframe during boost.

Since all three materials produce very high factors of safety under the assumptions, static compressive strength is not the limiting design constraint for the centering ring. Instead, other factors become more important in material selection:

- **Temperature tolerance.** PLA begins to soften at relatively low temperatures (around 60–65 °C), which could be reached near the motor casing during or after burn, especially if exhaust heating conducts through the structure. Aluminum and fiberglass maintain their mechanical properties much better at elevated temperatures.
- Manufacturability. PLA is easy to 3D-print directly to the desired geometry, enabling rapid prototyping and straightforward integration with other printed parts. Aluminum requires machining, but it is widely available and easy to work with in a machine shop. Fiberglass rings would likely be water-jet or router-cut from laminate panels, which requires different equipment

but allows high strength-to-weight ratios.

- Stiffness and long-term durability. While PLA meets the static strength requirement, it is more
 susceptible to creep and degradation under cyclic loading and repeated thermal exposure.
 Aluminum and fiberglass provide higher stiffness and better long-term durability for multiple
 flights.
- Integration with surrounding structure. If the aft end of the rocket is primarily composite or metallic, using fiberglass or aluminum for the centering ring can simplify bonding and reduce differential thermal expansion.

Overall, the analytical results confirm that the current geometry is structurally conservative for the H-100 motor, even for the weakest of the three candidate materials.

10. Conclusions and Recommendations

An analytical model of the motor centering ring was developed to evaluate its ability to carry the thrust of our H-class rocket motor. Using a conservative peak thrust of 613 N and modeling the ring as a compressed annulus, the maximum average compressive stress was found to be approximately 27 psi. For 3D-printed PLA, 6061-T6 aluminum, and fiberglass, the corresponding factors of safety with respect to yielding are 270, 1,490, and 2,640, respectively. A sensitivity study assuming a much smaller effective load area still yielded minimum factors of safety greater than 30.

From a purely static strength perspective, all three materials are more than sufficient for this component. Because of its ease of fabrication and good strength, PLA is suitable for prototype and ground-test rings. However, due to concerns about elevated temperatures and long-term durability, this memo recommends that the flight centering ring be manufactured from either 6061-T6 aluminum or fiberglass laminate, with a slight preference for aluminum due to its predictable isotropic behavior and straightforward machining.

These conclusions directly influence the project by justifying the use of a metal or composite centering ring for flight hardware and confirming that the current geometry does not need to be or redesigned for thrust loads. Future work could include a finite element analysis that includes the detailed slot geometry and adhesive joints, as well as thermal analysis to quantify the maximum temperatures the ring will experience during and after motor burn.

11. References

- [1] C. Pavon *et al.*, "Mechanical, Dynamic-Mechanical, Thermal and Decomposition Behavior of 3D-Printed PLA Reinforced with CaCO3 Fillers from Natural Resources," *Polymers*, 2022
- [2] "Aluminum 6061-T6 ASM material data sheet," MatWeb, accessed Nov. 2025 https://asm.matweb.com/search/specificmaterial.asp?bassnum=ma6061t6
- [3] "Overview of materials for Epoxy, Molded, Glass Fiber Filler," MatWeb, accessed Nov. 2025 https://www.matweb.com/search/datasheet.aspx?MatGUID=035d2130b7f34d918e8d590659b85 cb7