Below-the-Knee Exoskeleton

Engineering Calculations Summary

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1.1 Top Level CAD

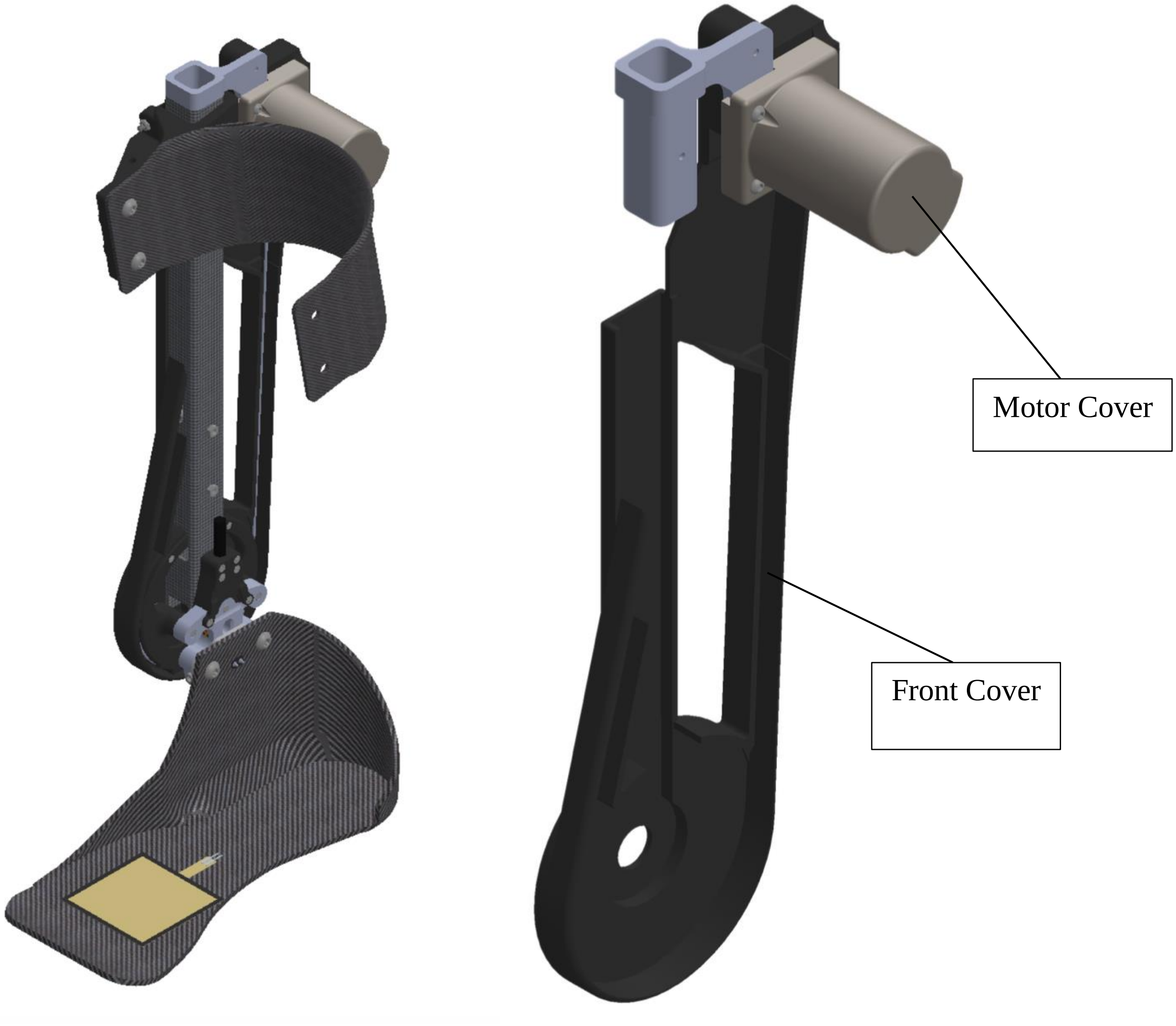


Figure 1: Full Assembly/Updated parts

Above is the current full CAD model, and the assembly is composed of only the updated parts. Included in the updated model are the motor subassembly, the motor cover, and the front cover. The front cover was updated to make room for the larger motor, while the motor cover was designed to fully protect the motor assembly.

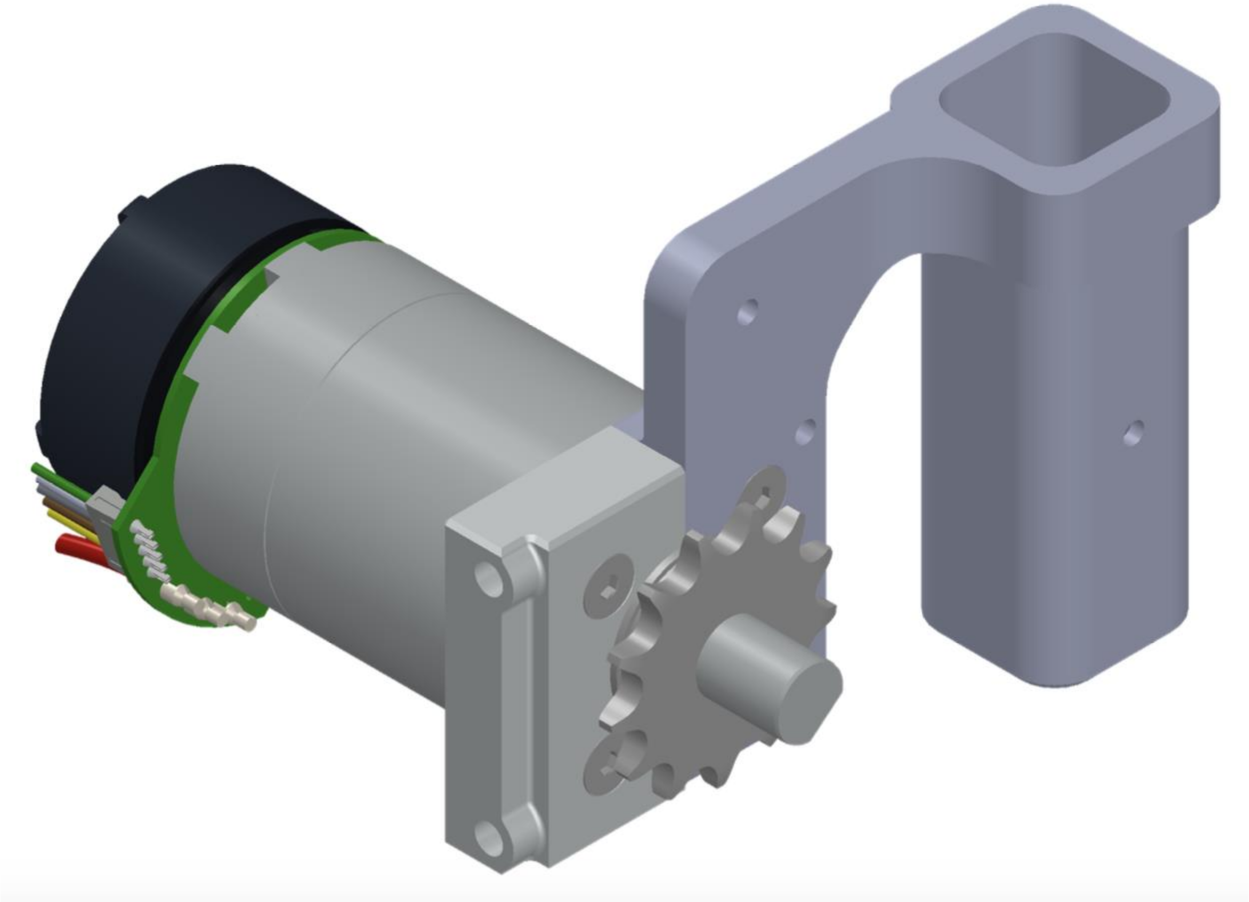


Figure 2: Motor Assembly

The motor assembly was fully redesigned because the new motor had a larger diameter, as well as a larger axis. The chain sprocket has the same outside diameter, but a larger axle diameter, and both the motor bracket and motor mount needed to be remade to fit the larger motor face.

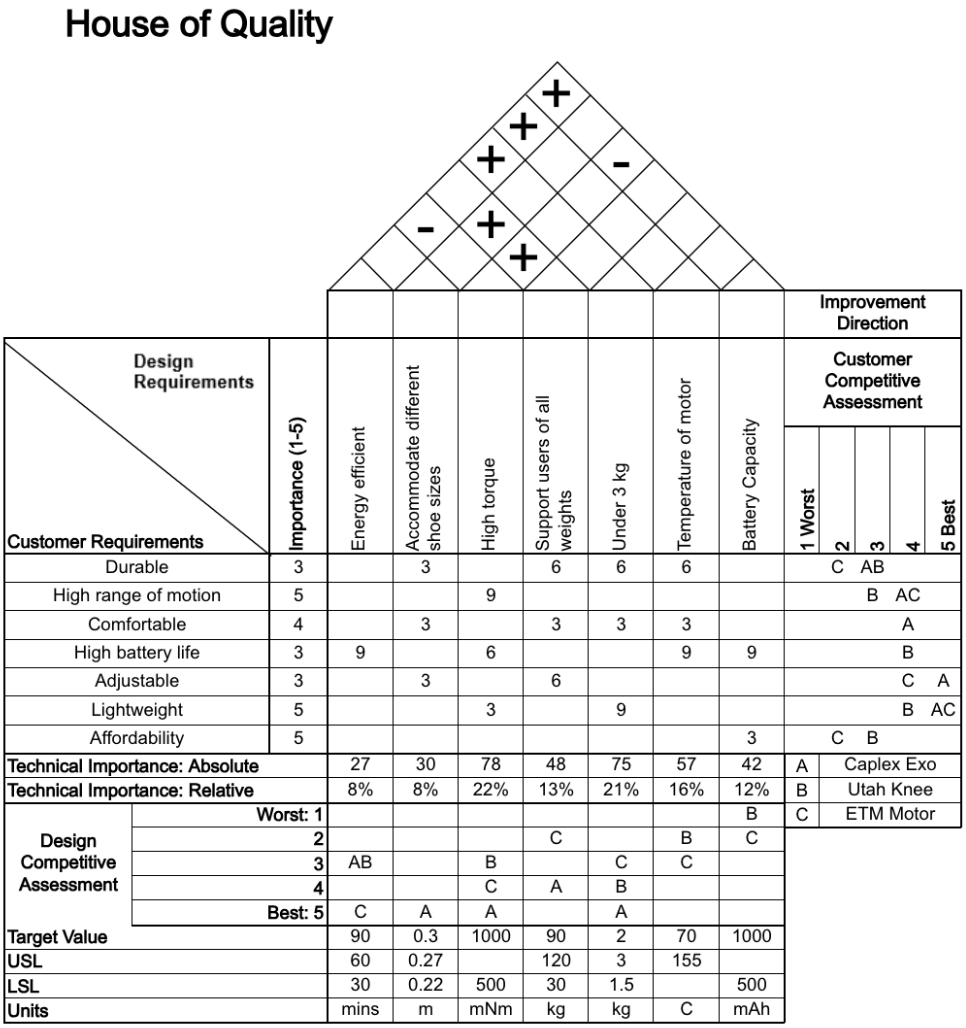
1.2 Updated QFD

1.2.1 Customer Requirements

The first customer requirement is durability. This will be important as the device will eventually be used in a customer’s everyday life, and when walking outdoors, it is likely that eventually they may either kick or walk too close to an object and contact the device. Eventually, we will design a protective cover, but along with this, the components need to be stable and close to the leg to avoid damage. A high range of motion is important as the device is designed to be assistive rather than simply an ankle brace. This means the user can use the motion of their ankle freely, and the device should be able to accommodate that full range of motion. Once again, the device will eventually be used for everyday life such as hiking and recreational walking. Both adjustability and affordability are important as the device is intended to be accessible to a wide range of individuals who need assistance. Because of this, the device is intended for those with cerebral palsy and other muscular deficiencies, it is crucial that the device is lightweight and can be easily attached to the user's feet for as long as they need while still allowing them to walk normally.

1.2.2 Engineering Requirements

The most important engineering requirement was determined to be the torque produced by the drive system. The new motor to be installed produces a nominal torque of .103 Nm and is configured with a 35:1 reduction planetary gearbox, equaling a torque of 3.6 Nm. The system is designed to assist users with muscular deficiencies, namely cerebral palsy, and as a rehabilitative device rather than a prosthetic, the device does not necessarily need to support the full weight of the user. A high torque will, however, allow the device to be operated by a wider range of users of greater weights or less leg strength. Next in importance is the device's weight. Again, with the target customer being users with cerebral palsy and muscular deficiencies, the device must be lightweight enough to be used without hindrance, and to be used for extended periods. We estimated the device should be no more than 3 kg so as not to negatively affect the user's gait. With a relative technical importance of 16%, the temperature of the motor needs to be low enough so as not to cause discomfort or injure the user. The motor is mounted inches from the user's leg, right behind the calf cuff so a motor running hot could easily affect the user. Aside from injuring the user, an overheating motor could also cause permanent damage to the exoskeleton frame and other electrical components nearby. The motor will be enclosed in a case both to prevent damage to the motor and to contain possible hot temperatures produced. The ability to accommodate users of all weights and sizes are similar requirement, as the device is a rehabilitative tool, the exoskeleton should have the capacity to be used by any client who needs it. As stated, supporting users of all weights is a function of the torque system and has been improved with a new motor. Accommodation for users of varied sizes relies on the frame. The footplate and calf cuff are modular for users with different foot and calf sizes. The cuff mount on the current design is designed to be vertically adjustable, however there is only room to move about an inch. We are currently designing a system to expand the adjustability of this component. The battery capacity will be more important in the future, however, as of now the device is intended for lab use and the current run time of about 30 minutes is sufficient. The new motor we will install has a capacity of 910 mA and will run for long enough while testing. Energy efficiency is the least important requirement, as stated, the current design is intended for lab use and the battery life does not need to be more than 30 minutes. In the future, a battery with a higher capacity will be selected so that the exoskeleton can be used daily. Figure 1 demonstrates the above requirements in a graphical form of the House of Quality.



House of Quality

Customer Requirements \ Design Requirements	Importance (1-5)	Energy efficient	Accommodate different shoe sizes	High torque	Support users of all weights	Under 3 kg	Temperature of motor	Battery Capacity	1 Worst	2	3	4	5 Best
Improvement Direction													
Durable	3		3		6	6	6			C	AB		
High range of motion	5			9							B	AC	
Comfortable	4		3		3	3	3					A	
High battery life	3	9		6			9	9				B	
Adjustable	3		3		6							C	A
Lightweight	5			3		9						B	AC
Affordability	5							3		C	B		
Technical Importance: Absolute		27	30	78	48	75	57	42					
Technical Importance: Relative		8%	8%	22%	13%	21%	16%	12%					
Design Competitive Assessment – **Worst: 1**								B					
2					C		B	C					
3		AB		B		C	C						
4				C	A	B							
Best: 5		C	A	A		A							
Target Value		90	0.3	1000	90	2	70	1000					
USL		60	0.27		120	3	155						
LSL		30	0.22	500	30	1.5		500					
Units		mins	m	mNm	kg	kg	C	mAh					

Customer Competitive Assessment

A	Caplex Exo
B	Utah Knee
C	ETM Motor

Figure 3: QFD

2 Summary of Equations and Solutions

2.1 Equations

2.1.1 Mathematical Modeling of the Foot – Nick Watkins: The first calculation was to model the forces of the foot at the location where the force is the greatest. We assumed a weight of 200 lbs (90kg) and a shoe size of 10.5. Figure 3 demonstrates the free-body diagram of the forces below the ankle. We were able to calculate ground force, 1068N, force at the ball of the foot, 958N, and the work, -45.12J. Figure 4 demonstrates the method with which we found the above values. The calculations for work were then used for the next set of calculations, 3.3.2. These calculations aided in the team's research into the ideal motor for the prosthetic.

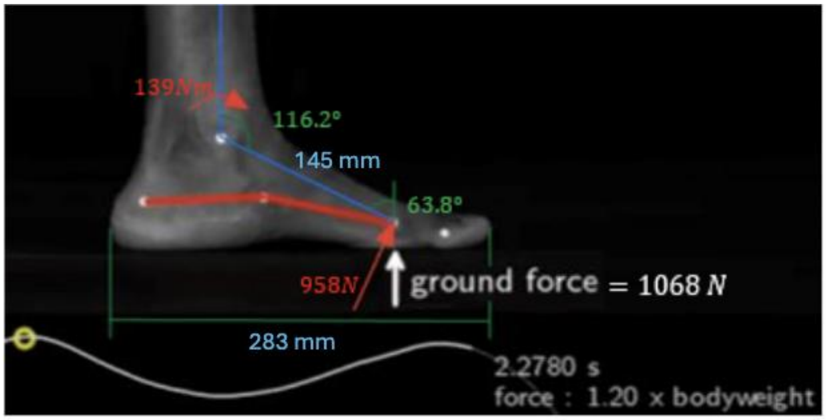
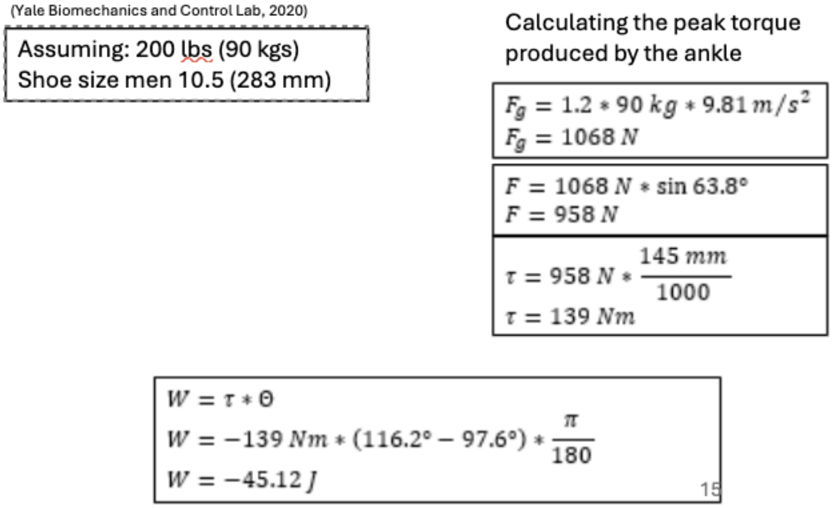


Figure 2: Free Body Diagram


(Yale Biomechanics and Control Lab, 2020)

Assuming: 200 lbs (90 kgs)
Shoe size men 10.5 (283 mm)

Calculating the peak torque produced by the ankle

$$F_g = 1.2 * 90\ kg * 9.81\ m/s^2$$
$$F_g = 1068\ N$$

$$F = 1068\ N * \sin 63.8°$$
$$F = 958\ N$$

$$\tau = 958\ N * \frac{145\ mm}{1000}$$
$$\tau = 139\ Nm$$

$$W = \tau * \Theta$$
$$W = -139\ Nm * (116.2° - 97.6°) * \frac{\pi}{180}$$
$$W = -45.12\ J$$

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Equation 1: Mathematical Modeling Calculations

2.1.2 Torque Output by Motor onto Motor Mount – Ryan Oppel: For this Analysis of the motor mount, what we are looking to find is the stresses applied from the motor to the mount and all other components. We are working with an ECX FLAT Maxon motor so to start with this analysis we must define the amount of force that this motor will have on the mount. Our mount for the motor consists of two parts that will be screwed onto the motor, a carbon fiber tube, and each other. These two parts that create the mount of the motor will also hold a cover for the motor which will also be screwed into each other. Another variable important to know when doing a stress analysis is the type of material used for the mounts and other components attached to the motor or mounts. For these months, our client, Dr. Lerner picked out the material that we will use. The material for these mounts is 7075 - O (ss) Aluminum alloy. This is the type of material that he has used in the past for his exoskeleton projects, and it has shown that this material is the perfect balance between strong, lightweight, and thermally conductive which will help us with thermal management of the motor later down the line. The last criterion to understand to perform a stress analysis will be the physical dimensions itself. Because our parts are dimensionally complex, I will be using SolidWorks software to visually show the stresses on the material. I believe it is important to bring up this variable because, in the design phase of the project, the team made sure that we designed parts to have as little of a weak point as possible, we did this by filleting holes and edges and creating connecting parts where we can ensure structural integrity. While we are doing some of these calculations through SolidWorks, we are still doing many by hand. In this analysis, we will ultimately find the factor of safety of our motor mount (FoS). In this, we must calculate many things starting with the force the motor will displace onto the mount. Since the motor has an attached gearbox, we will be finding the torque in which that gearbox will apply. Then we must find the mounting points on the mount and find the forces that will be applied to each of them. After that, we will find the weakest point in the mount and do a force analysis there where we can calculate our factor of safety with our known material.

To start with our calculations, I must first find the amount of torque that the motor will apply to the motor mount. I found this information on the manufacturer's website. For the ECX FLAT Maxon motor, we find that the nominal torque that this motor will provide at maximum power will be 103 mNm. With this information, I will then calculate how much torque will be applied to the motor mount by using a simple gear-ratio equation with our gearbox which is a 1:35 gear ratio.

$$\text{Output Torque} = \text{Input Torque} \times \text{Gear Ratio}$$

$$\text{Output Torque} = 103\,\text{mNm} \times 35 = 3605\,\text{mNm}$$

$$3605\,\text{mNm} = 3.605\,\text{Nm}$$

Equation 2: Torque Output Equation

With this value, I must conceptually understand how these forces will be applied to the mount itself. Now that I know the value at which the motor is applying torque to the rest of the mount, I must find out what points are being displaced in the mount, find their distance from the displaced torque, and how many points of contact there are.

The motor is mounted to the bracket by four screws that are 3mm in diameter and are situated exactly 13mm away from the center of the motor shaft. With this information, I can calculate how much force each mounting point will experience with a simple torque equation.

$$T = F \cdot r \implies F = \frac{T}{r}$$

$$T = 3.605\,\text{Nm} \qquad F = \frac{3.605}{0.013} = 277.31\,\text{N}$$

$$r = 13\,\text{mm} = 0.013\,\text{m}$$

Equation 3: Torque Stress Equation

With this number, I then divided the total force by 4 to find out how much each mounting force would experience which gave me a final number of 69.33N. The only other stresses applied to the mounts are the downward force of the weight of the motor cover and the mounted motor which is attached to a chain which came out to approximately 2N of force.

Now with the torques known on each mounting point for the motors to mount onto the brackets, I then did a finite element analysis in SolidWorks of all the forces applied to the motor mounts.

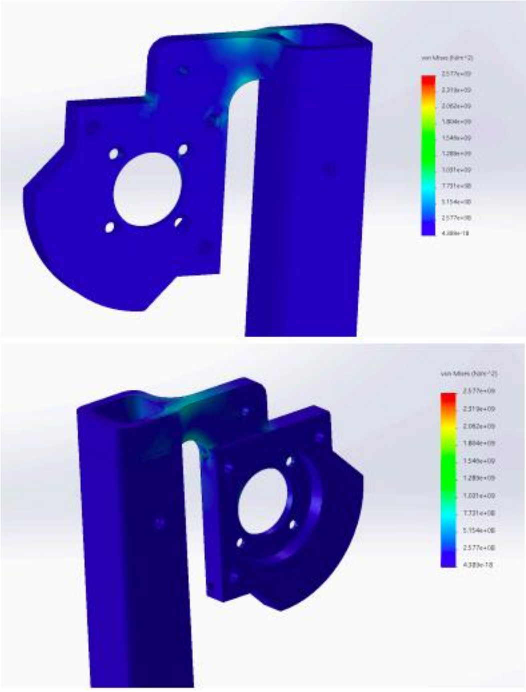


Figure 4-5: visuals of the FEA applied to the motor mounts with known variables

As we can see, where the connection is between the motor mount and the carbon fiber tube, the design comes to a bottleneck which is where we can see where our biggest calculated force peaks. According to SolidWorks, the max amount of stress that was applied to our design was 1.042*10^7 N/m^2 which comes out to approximately 10.42MPa.

According to the ASM Material data sheet, 7075 - O (ss) Aluminum alloy has a yield strength of approximately 145MPa. With these new values, I can now calculate the factor of safety for our motor mounts.

The equation for the Factor of Safety is:

$$\mathrm{FOS} = \frac{\sigma_{\text{yield}}}{\sigma_{\text{applied}}}$$

$$\mathrm{FOS} = \frac{145}{10.42} \approx 13.91$$

Equation 4: Factor of Safety

$$T_{\text{output}} = T_{\text{input}} \times \text{Gear Ratio}$$

Equation 5: Torque Output Calculation

$$\text{Total Efficiency} = (\text{Stage Efficiency})^{\text{Number of Stages}}$$

If we assume a 2% loss per stage and a multi-stage gearbox with 3 stages to achieve the 89:1 ratio:

$$\text{Total Efficiency} = (0.98)^3 \approx 0.94$$

Equation 6: Total Efficiency Calculation

Overall, these calculations allowed the team to analyze whether the current design for the base could still support the motor at full torque, if the base needed to be reanalyzed, or if a new location for the motor needed to be considered. Due to the factor of safety being within a reasonable realm, the team determined no redesign needed to be considered for the main shaft or location of the motor.

2.1.3 Stress on the shaft of the Motor – Alex Schell: If torque output is labeled as the torque needed at the ankle and torque input is measured at the motor, both calculations were solved in the previous presentation. Nominal torque from the motor at maximum power is 103 mNm due to the specs of the motor. The motor also has a gear ratio of 1:35. With the equations listed in equation 7 the torque being 103 mNm and the radius of the shaft, as designed in SolidWorks, being 3 mm, the stress is calculated at 65.6 MPa. This calculation allowed the team to ensure the structural integrity of the motor and to ensure the shaft would hold up at full power of the motor. This calculation can be furthered in future calculations by analyzing the effect of this stress on the chain connecting the motor to the footplate.

Stress at the motor

$$\tau = \frac{T*r}{J}$$

τ = Stress
T = Torque
r = radius of the shaft

$$J = \frac{\pi r^4}{2}$$

J = Polar moment of inertia

Equation 7: Stress Calculations

2.4.1 SolidWorks Simulation – Alex Schell: A simulation in SolidWorks was performed to show the stresses and the life cycle of the current motor mount, which is one of the items we are tasked with redesigning. The current location is at the back of the model, prone to being bumped into surrounding objects. There is no water resistance or protection against debris, which is another customer requirement. The current placement is prone to high stress and fracture. Figure 6 demonstrates the Von Mises stresses based on a force of 15N. Figure 7 demonstrates the Life Cycle based on 15N after 1000000 cycles. This calculation allowed the team to analyze where the faults lie in the current design and use this knowledge when looking at other options. We eventually went with a fully enclosed mount, that allows for more strength along the length of the mount and lowered the stresses. The new design, portrayed in Chapter 1.1, also allows for more protection against heat and debris.

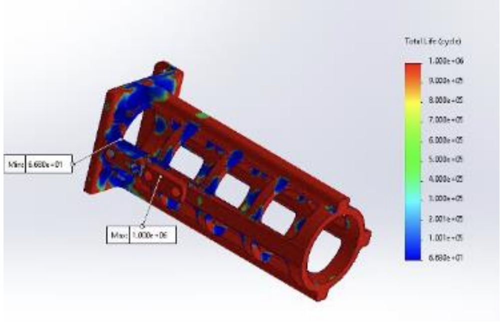


Figure 6: Von Mises Stress

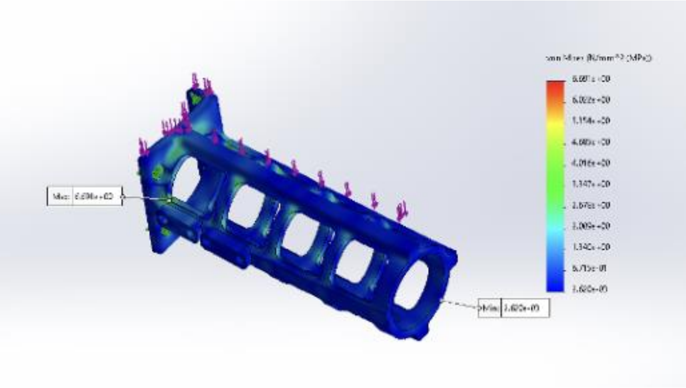


Figure 7: Life Cycle

2.1.5 SolidWorks Simulation – Alex Schell: I decided to run an analysis of the thermal dispersal from the motor to the edge of the cover. One of the main challenges our team faces is preventing heat from the motor from being felt by the user via the dispersal of the heat using a motor cover. This can happen either from the material used or by the usage of heat sinks or a fan. Our current design puts some heat sinks around the perimeter of the cover, using aluminum as the material. While we are looking at a redesign of the current location of the heat sinks, the current thermal analysis is done on the current design.

In this analysis, the main equation used was to determine heat emanating from the motor's coils and the convective heat transfer to the surrounding air. To solve the heat dissipation of the motor, we used formula (1). [1]

$$(1)\ \ H_{dissipated} = P_{input} * (1 - efficiency)$$

P_{input}, or the input power, is the next calculation that is needed to find heat dissipation. Formula (2) shows the power input formula. Input power is based on efficiency and output power. [1]

$$(2)\ \ P_{input} = \frac{P_{output}}{efficiency}$$

Efficiency was calculated by one of my teammates in an earlier calculation, assuming 82.7% efficiency. Formula (3) calculates output power using torque (T) and angular velocity (). [2]

$$(3)\ \ P_{output} = T * \omega$$

The assumption from past calculations was that the torque produced from the motor is 2.8 Nm. The next calculation that had to be completed was to find the angular velocity. Assuming a nominal speed, of 184.3 rpm, I used formula (4) to find angular velocity.

$$(4)\ \ \omega = \frac{2\pi * RPM}{60}$$

Using the above formulas, I calculated angular velocity to be 19.30 rad/sec and the power output (P_{output}) to be 54.04 W. Using this value, we can now calculate input power (P_{input}) to be 65.34 W and total dissipated heat ($H_{Dissipated}$) to be 11.3 W.

From here, I used the existing designs created by Ryan to run a thermal study in a steady state, using calculations for the convective heat transfer coefficient using formula (5), where L is the characteristic length, which is the height or 58mm.

$$(5)\quad h = \frac{Nu * k_{air}}{L}$$

Formula (5) uses Nusselt Number (Nu) which can be calculated using formula (6) I used the assumption for this formula that the Prandtl number (Pr) is 0.715, assuming 25 ℃. [3]

$$(6)\qquad Nu = 0.3 + \frac{0.62 Pr^{1/3}}{[1+(0.4/Pr)^{2/3}]^{1/4}} Re^{1/4}[1 + \left(\frac{Re}{282000}\right)^{\frac{5}{8}}]^{4/5}$$

To calculate the Reynolds number (Re), I used the formula (7). For this formula, I used the following assumptions: density (ρ) is 1.18 $\frac{kg}{m^3}$, flow velocity (u) is 5 m/s, dynamic viscosity (μ) is 1.64E-5 $\frac{kg}{m*s}$, thermal conductivity (k) is 0.026 $\frac{W}{m*K}$, and again, the Prandtl number (Pr) is 0.715. [3]

$$(7)\quad Re = \frac{\rho * u * L}{\mu}$$

Formulas 5-7 were found in a document from the Microelectronics Heat Transfer Laboratory. [4] Since the Reynolds number (Re) was equal to 18,598, which is under 500,000, this example is laminar flow. Using that number and formula (6), the Nusselt number (Nu) was calculated to be 86.08. Using the formula (7), I calculated that the convective heat transfer coefficient was 38.59$\frac{W}{m^2K}$.

Overall, based on the above equations, I assumed heat production was 11.3 W, while the convective heat transfer coefficient at 298 K was 38.59 $\frac{W}{m^2K}$. Using these values, I ran a SolidWorks evaluation on the heat transfer across the motor. The current model has heat sinks surrounding the motor cover. Figure 1 shows the heat distribution of the current motor cover design.

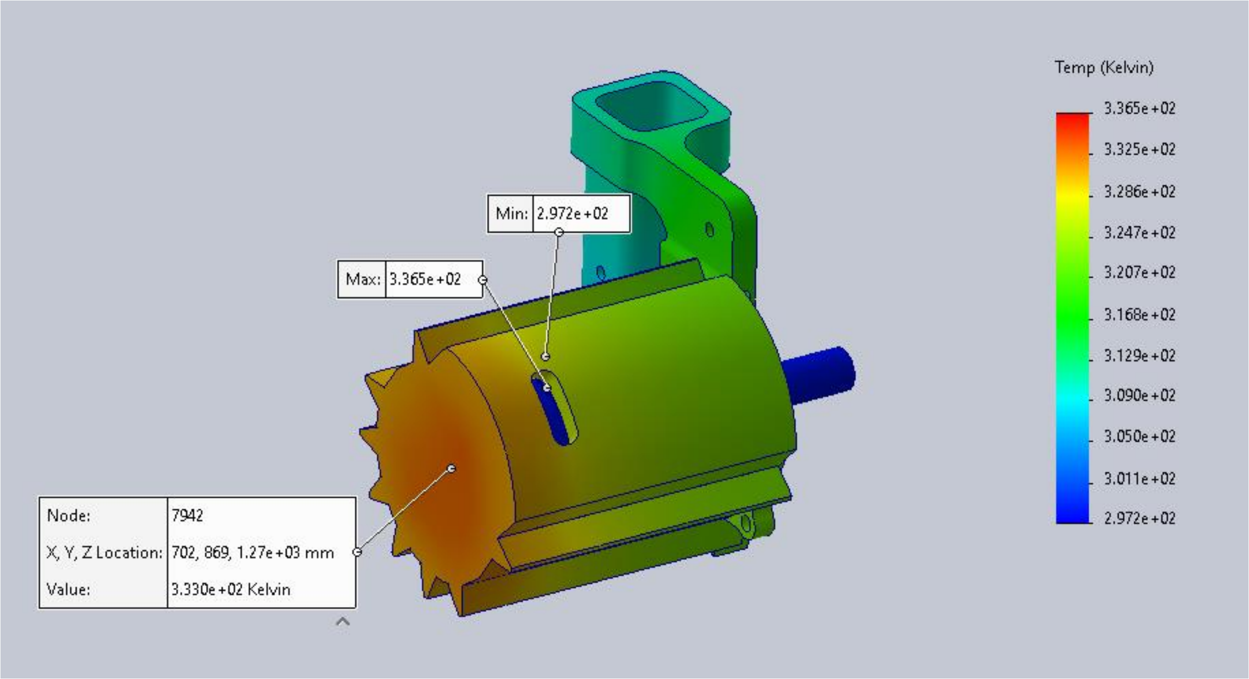


Figure 1: Thermal Analysis of motor cover

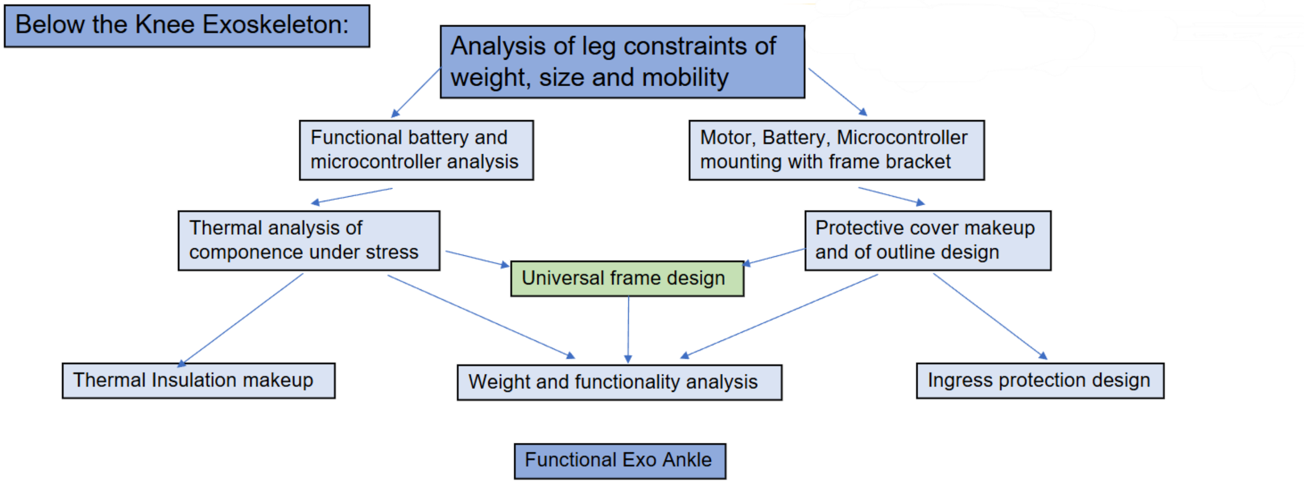
We determined heat sinks were the most efficient way to disperse heat across the cover. While the current cover disperses heat well, a redesign could be done in the future to improve the heat dispersal, especially as one edge of the cover remains hot. My team was discussing placing the heat sinks on top of the motor rather than over the cover, acting as a barrier between the cover and the motor. On top of improving the dispersal of heat, the newly proposed redesign would also improve comfortability.

2.2 *Minimum FOS*

Sub-system	Part	Load Case Scenario	Material	Method of calculating FoS	Minimum FoS
Battery-PCB Box					**4.30**
	Upper case	50 lbf applied to the center point of the carbon plate	Carbon fiber reinforced PLA	FEA in Solidworks	4.30

	Lower case	25 lbf applied to the center axes of the carbon backing	Carbon fiber reinforced PLA	FEA in Solidworks	8.70
Motor Mount					**13.91**
	Motor Bracket	Expected a yield force of 10.72MPa from the motor	7075 - O (ss) Aluminum alloy	FEA in Solidworks	14.75
	Motor Mount	Expected a yield force of 10.72MPa from the motor	7075 - O (ss) Aluminum alloy	FEA in Solidworks	13.91

3 Flow Charts and Other Diagrams



This diagram is a physical decomposition of our design and its constraints. This diagram is to show how we need to conceptually design our project. There are three main constraints that we need to take into consideration when designing our exoskeleton. As shown from the left branch our design must involve some sort of thermal management. Our task to bring all the electrical components to the foot means that we need to find a way to keep these components cool without causing any discomfort to the user. As shown by the right branch of the diagram we need to design a way to protect these parts from the surrounding environment. This will be difficult because this design constraint will be working in opposition to thermally managing all the electrical components. With both these design constraints in mind we also must keep the weight of the design down as much as we can as shown by the middle branch. Our task is to properly create a design that will fit each of these opposing design constraints as shown by the diagram.

4 Moving Forward

From here on out, our biggest concern is trying to build the designs we have in CAD. We need to get confirmation from our client that our designs are up to standard and confirmation that we can proceed with the development stage of the project. Beyond the production stage of our system, the other aspect of our next steps is the testing aspect. The main goals of our project are the mounts and casings for the battery, motor, and PCB. One of the considerations that goes into these designs is the protection against debris and thermal damage. As of right now, we have a SolidWorks simulation on the heat dispersal of the motor mount, but we have yet to complete a thermal analysis on the PCB and battery covers. That is the next step for the team. To use a simulation to prove our design disperses the heat production sufficiently. After this is complete, we will need to analyze how our designs protect against debris and light water contact. The team will need to learn how to test this aspect of the design in a simulation form and in-person testing. These simulations will include additional calculations to aid in the values needed to complete the simulations, such as a recalculation of the equations from 2.1.5 for the other two casings.