Department of Mechanical Engineering
ME 486C
1/26/24
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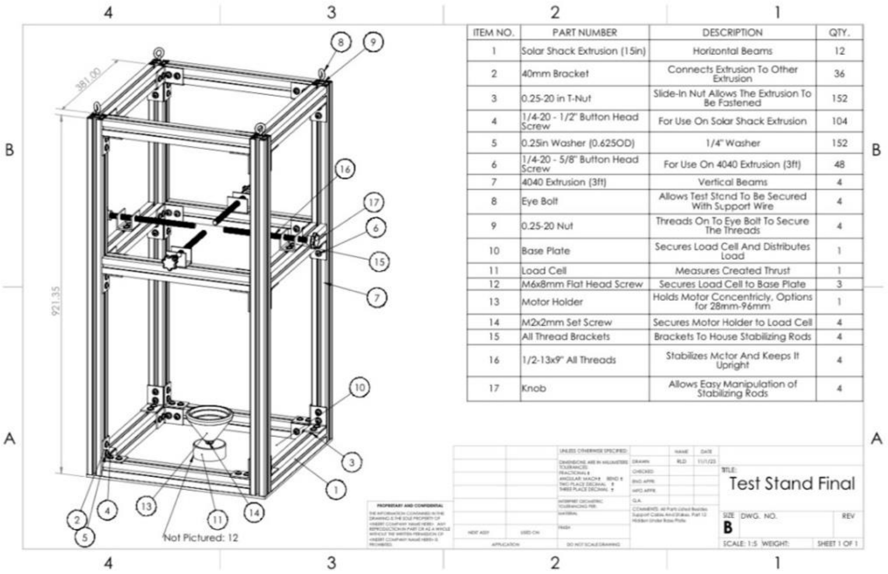
ME486C - Engineering Calculations Summary

Top Level Design Summary

In this paper, the team will define the projects current calculations relating to factors of safety for each component of the design and discuss the engineering calculations and sources which were used to determine these. The subsystems that will be evaluated through engineering calculations include the test stand, motor casing, nozzle, and rocket motor (propellant).

Test Stand

The test stand shown in figure 1 serves the purpose of measuring thrust data for motor testing. It is equipped with a load cell that captures thrust data, subsequently inputting it into rocket motor software like burn sim. This process enables the team to obtain the burn curve and impulse of the motor. The test stand will also be used for evaluating various nozzle designs and conducting tests on a full-size, 75 mm motor, prior to its integration within the rocket.



ITEM NO.	PART NUMBER	DESCRIPTION	QTY.
1	Solar Shack Extrusion (15in)	Horizontal Beams	12
2	40mm Bracket	Connects Extrusion To Other Extrusion	36
3	0.25-20 in T-Nut	Slide-In Nut Allows The Extrusion To Be Fastened	152
4	1/4-20 - 1/2" Button Head Screw	For Use On Solar Shack Extrusion	104
5	0.25in Washer (0.625OD)	1/4" Washer	152
6	1/4-20 - 5/8" Button Head Screw	For Use On 4040 Extrusion (3ft)	48
7	4040 Extrusion (3ft)	Vertical Beams	4
8	Eye Bolt	Allows Test Stand To Be Secured With Support Wire	4
9	0.25-20 Nut	Threads On To Eye Bolt To Secure The Threads	4
10	Base Plate	Secures Load Cell And Distributes Load	1
11	Load Cell	Measures Created Thrust	1
12	M6x8mm Flat Head Screw	Secures Load Cell to Base Plate	3
13	Motor Holder	Holds Motor Concentricly, Options for 28mm-96mm	1
14	M2x2mm Set Screw	Secures Motor Holder to Load Cell	4
15	All Thread Brackets	Brackets To House Stabilizing Rods	4
16	1/2-13x9" All Threads	Stabilizes Mctor And Keeps It Upright	4
17	Knob	Allows Easy Manipulation of Stabilizing Rods	4

Figure 1. Motor Test Stand Assembly

Motor Casing

The motor casing design shown in figure 2 contains and aligns the propellant, ensuring its stability while being burned on the test stand. Adhering to Tripoli Rocketry Safety Standards, the casing must be crafted from a non-ferrous ductile material, mitigating the risk of sparks or fragmentation in the event of casing failure.

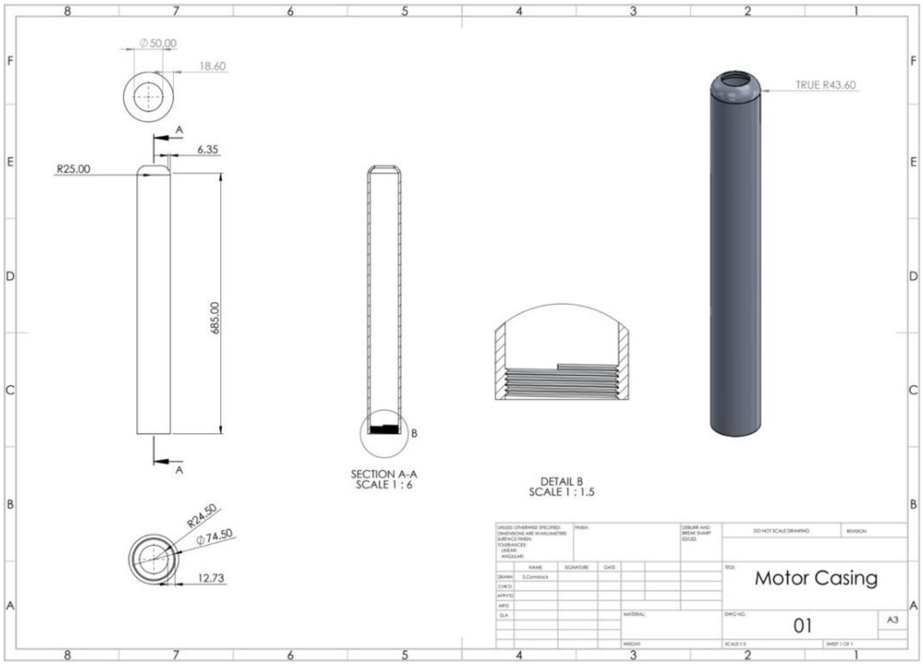
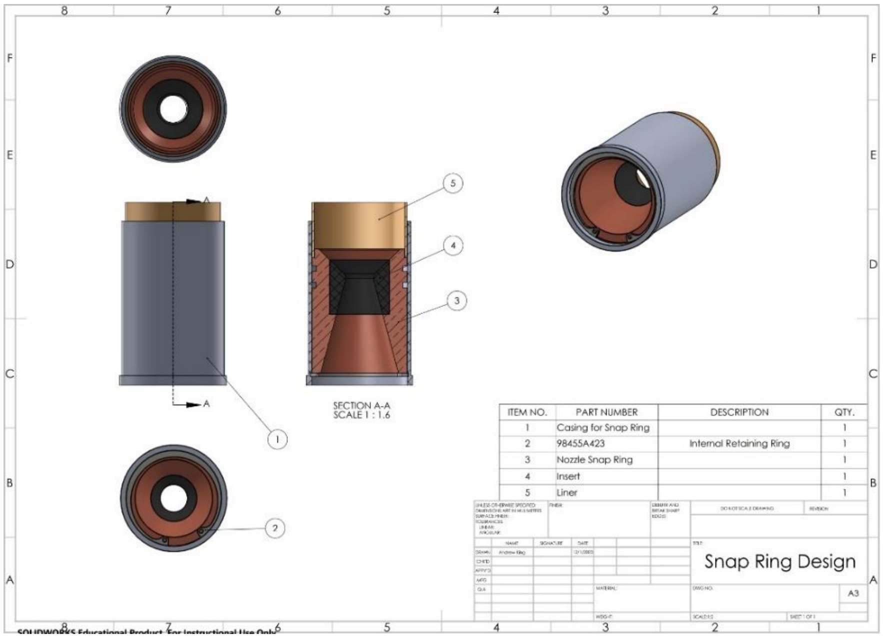


Figure 2. *Rocket* Motor Casing

Nozzle

The rocket derives its thrust from the nozzle, and the converging-diverging segments of the nozzle are designed to achieve optimal thrust ratios. The team is currently designing the nozzles material to be a phenolic resin with a graphite insert, shown in figure 3 below. The nozzle's suitability for the application will be determined through experimental trials on the test stand. These engineering calculations aim to identify the most effective nozzle configuration for the project's requirements.

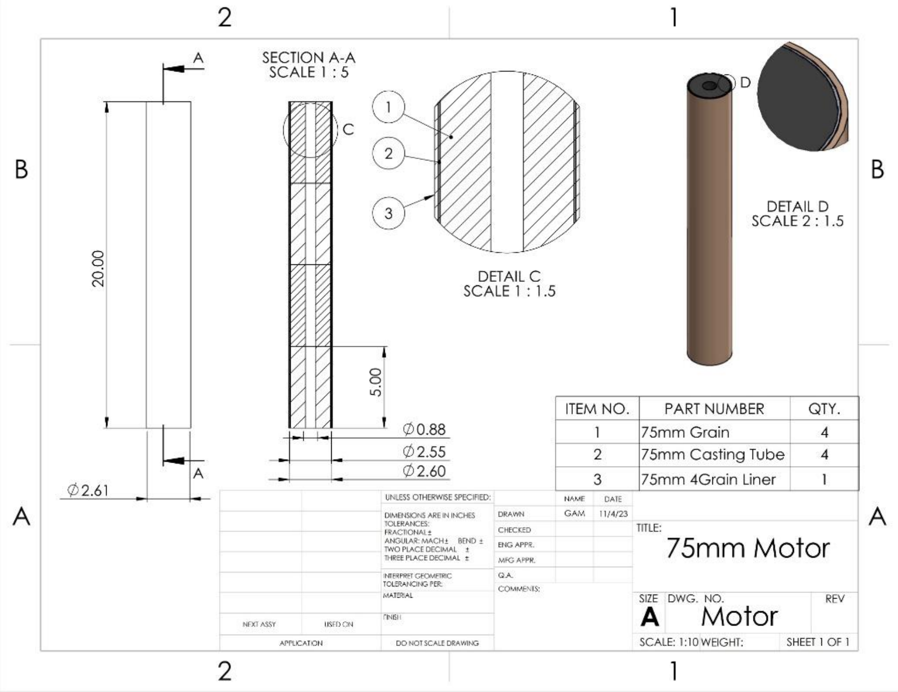


ITEM NO.	PART NUMBER	DESCRIPTION	QTY.
1	Casing for Snap Ring		1
2	98455A423	Internal Retaining Ring	1
3	Nozzle Snap Ring		1
4	Insert		1
5	Liner		1

Figure 3. Rocket Nozzle Assembly

Propellant

The propellant serves as the rocket's fuel in order to create its propulsion, making this component especially crucial in the success of the rocket. Specifically, the propellant utilizes an Ammonium Perchlorate Composite with aluminum powder as its fuel. The grain geometry employed in this propellant is identified as BATES grains (Ballistic Test and Evaluation System), due to this geometries simplicity and efficiency. The BATES grain is shown in figure 4 below, which is simply a donut shaped cross section geometry throughout the motor.



ITEM NO.	PART NUMBER	QTY.
1	75mm Grain	4
2	75mm Casting Tube	4
3	75mm 4Grain Liner	1

Figure 4. Rocket Motor Assembly

Throughout this report, the team will define the necessary parameters

Summary of Standards, Codes and Regulations

To ensure the safety and legality of this project, the team is adhering to rules and regulations set in place by Tripoli Rocketry and the Federal Aviation Association (FAA). Since the set of rules from both organizations is so expansive, the team has tabulated the ones relevant to our project in Table 1 and Table 2. See Appendix A for the extended table. The code provided by Tripoli Rocketry only governs the actions of anyone at a Tripoli Launch but will be followed by our team regardless of the location of our launch. The laws

Table 1. Relevant Tripoli Safety Codes

Tripoli Rocketry Safety Code	
Code #	Description
7-1.4	The thrust-to-weight ratio of a rocket typically should be at least 5:1. However, the RSO may approve a thrust-to-weight ratio as low as 3:1 ratio.
7-2	The rocket must remain pointed in a safe direction (away from all people) while installing the igniter and using the igniter then after.

7-8	The motor igniter shall not be connected to the launch system until all other flight electronics are active.
9-4	Rockets shall not be launched in any way that could interfere with aircraft operations.
9-6	No rocket shall be intentionally launched into or through a cloud.
10-10.1	Metallic cases shall be made of non-ferrous ductile metals such as 6061 aluminum alloy.
10-10.3	Forward closures shall not be made of ferrous materials.
10-10.4	Minor components such as snap rings, nozzle washers, rear closures, and seal disks may be made of ferrous materials.
13-6	A high-power rocket must be launched no more than 20 degrees from vertical.
13-7	Igniters shall not be installed in high power rocket motors except at the pad or at special preparation area away from all uninvolved people.

Table 2. Relevant FAA Laws

FAA Laws	
Code #	Description
7400.2, Section 2, 31-2-2. a. 4.	In accordance with part 101, an amateur rocket must not create a hazard to persons, property, or other aircraft.
7400.2, Section 2, 31-2-2. b. 1.	Class 2-High Power Rockets and Class 3-Advanced High-Power Rockets must not operate at any altitude where clouds or obscuring phenomena of more than five-tenths prevail
7400.2, Section 2, 31-2-2. b. 2.	Class 2-High Power Rockets and Class 3-Advanced High-Power Rockets must not operate at any altitude where the horizontal visibility is less than five miles

Summary of Equations and Solutions

To determine any potential modes in which the team's design components could fail, a failure mode and effects analysis (FMEA) was performed for each of the project's subsystems. The subsystems were categorized as test stand, rocket motor, motor casing, and nozzle. In the following section, the team will show the Failure Modes and Effects Analysis (FMEA) for each of these subcategories, accompanied by an analysis of the insights derived from the FMEA process.

Test Stand

Table 3. FMEA for Test Stand

Product Name: Motor Test Stand	Development Team: Shannon Comstock, Remy Dasher, Andrew King, Grace Morris	Page No 1 of 1

System Name: Motor Testing			FMEA Number
Subsystem Name: Thrust and Impulse Analysis			Date 1/26/2024
Component Name:	Test Stand		

Part # and Functions	Potential Failure Mode	Potential Effect(s) of Failure	Severity (S)	Potential Causes and Mechanisms of Failure	Occurrence (O)	Current Design Controls Test	Detection (D)	RPN	Recommended Action
Top Brackets	Temperature Deformation	Top extrusion bars and brackets will loosen due to expansion from motor exhaust heat, decreasing the structural integrity over time	3	Motor exhaust reaches high temperature	4	Maintenance requirement to tighten all bolts	2	24	None
	Thermal Fatigue from the expansion and contraction from the exhaust heat on the brackets	Crack propagations can cause brackets to fail, decreasing the structural integrity	4	Motor exhaust reaches high temperature	2	Coat/ cover the parts exposed to heat with thermal shielding	3	24	None
Bottom Brackets	Force Deformation	The brackets warp causing the structural integrity to decrease.	2	Impact loading from thrust force	2	Designed to withstand max loading	3	12	None
Top Extrusion Bars	Temperature Deformation	The aluminum extrusion warps from heat exposure, causing structure to deform and potentially effect grip on motor	6	Motor exhaust reaches high temperature	5	Coat/ cover the parts exposed to heat with thermal shielding	4	120	Simulate thermal analysis of these components and determine if design needs to be altered
	Thermal Fatigue from the expansion and contraction from the exhaust heat	The aluminum extrusion fractures/ fails, causing motor to lack support in its fixed position	3	Motor exhaust reaches high temperature	2	Coat/ cover the parts exposed to heat with thermal shielding	3	18	None

Bottom Extrusion Bars	Force Deformation	Instability in motor mounting during testing may cause motor to become dislodged and become a projectile	9	Impact loading from thrust force	4	Designed to withstand max loading	5	180	Simulate structural analysis of these components and determine if design needs to be altered
Bolts	Force deformation (Shearing)	Bolts shear during testing, potentially causing bracket to be unsupported	4	Impact loading from thrust force	5	Designed to withstand max loading	4	80	None
	Force Deformation (Normal Stress)	Bolts buckle from compressive stress causing lack in structural integrity	4	Impact loading from thrust force	5	Designed to withstand max loading	3	60	None
	Temperature Deformation	Bolts come loose from the structure, causing instability, potentially lack of support for motor	4	Motor exhaust reaches high temperature	2	Maintenance requirement to tighten all bolts	2	16	None
	Thermal Fatigue from the expansion and contraction from the exhaust heat	Bolts crack during testing, decreasing structural integrity	2	Motor exhaust reaches high temperature	2	Coat/ cover the parts exposed to heat with thermal shielding	2	8	None
T-Nuts	Force Deformation (Normal Stress)	Loosens from bolts, compromising the strength of the brackets hold on the extrusion	2	Impact loading from thrust force	1	Maintenance requirement to tighten all bolts	2	4	None

From this analysis, the most critical components related to the structure's failure were found to be the top and bottom extrusion bars. A design flaw of the test stand was discovered through this process where the top extrusion bars will be subjected to heat from the motors exhaust, causing the aluminum material to warp. If these extrusion bars warp during the burning of a rocket motor, the motor could lose stability in the bracket and eject out of the test stand. This would be an extremely dangerous failure, and therefore the team determined to modify the design to incorporate heat shieling around the top extrusion bar.

For the bottom extrusion bars, potential failure modes are linked to material fatigue induced by loading. Should these bars experience warping due to loading fatigue, the attached brackets connecting them to the overall structure become more susceptible to failure. In the event of separation between these bolts and brackets from the bottom extrusion bar, again the motor is at risk of dislodging, leading to a hazardous failure. Therefore, the team determined that mounting the motor holder and load cell to the base plate instead of the bottom extrusion bar would prevent the extrusion bar from excessive fatigue during loading.

Rocket Motor

Table 4. FMEA for Rocket Motor

Product Name		Development team: Shannon, Remy, Andrew, Grace				Page No 1 of 1			
System Name						FMEA Number			
Subsystem Name						Date 11/3/23			
Component Name	Motor								
Part # and Functions	Potential Failure Mode	Potential Effect(s) of Failure	Severity (S)	Potential Causes and Mechanisms of Failure	Occurrence (O)	Current Design Controls Test	Detection (D)	RPN	Recommended Action
Propellant Grain	Force Deformation	Propellant falls out of rocket either before or during combustion depending on severity, this leads to unsafe conditions for spectators and decreased altitude performance	9	Incorrect propellant formulation: Propellant is not fully cured	4	Hardness checks	2	72	Two or more team members should look over the grains prior to launch
	Accelerated Burn Rate	Pressure builds and the rocket explodes, or the fuel burns too fast and decreases altitude performance	10	Propellent contains voids due to errors in manufacturing processes	3	Density checks	2	60	Include a shake table in manufacture to reduce size and occurrence of voids
	Decelerated Burn Rate	Minimum thrust-to-weight ratio is not meet and rocket cannot lift off	7	Incorrect propellant formula	7	Iterative formula testing and analysis	1	49	Double check final iterations with a mentor prior to launch
Casting Tubes	Inconsistent Part Quality	The propellant grain does not fit properly in the liner and needs to be trimmed down	2	Casting tube is ripped during manufacturing	1	Rough quality control of incoming parts	2	4	None
Motor Liners	Inconsistent Part Quality	The motor does not fit properly in the casing	6	Incorrect Tolerances	1	Rough quality control of incoming parts	2	12	None

E-Matich	Short Circuit	Ignition would fail and nothing would happen	5	Manufacturer error	1	Rough quality control of incoming parts	6	30	None

Table 4 shows the potential failure modes for the propellant grains and rocket motor. The primary concern is that the propellant inside the motor could deform due to the forces it will be experiencing during launch. If the propellant inside the motor deforms or shears off this could block the nozzle exit. Blocking the nozzle exit would rapidly increase the pressure in the chamber, this pressure spike could cause the casing to rupture. If the propellant that shears off is too small to block the exit, then chunks of propellant will be ejected from the rocket while they are still burning. This could cause fires depending on where those pieces land and how quickly they burn up. This also increases spectator risk. To prevent this the team performed some calculations which will be shown below to ensure that the binder being used will be strong enough to prevent shearing. Additionally, the team will ensure all grains are fully cured before launch, meaning grains must be cast at least 3-5 days before launch as specified by the binder.

The secondary failure mode would be a rapidly accelerated burn rate, caused by voids and air pockets in the propellant grain. These voids are bubbles that are trapped in the propellant while it is curing. This would cause slight pressure spikes and decrease the maximum altitude that the rocket can reach since it is essentially lowering the burn time. To counteract this the team will use a shake table to help remove bubbles from the propellant. This preventative measure will decrease the number and size of any bubbles in the propellant grain.

Motor Casing

Table 5:FMEA for Motor Casing

Product Name	NAU Rocket Capstone	Development Team: Shannon Comstock, Remington Dasher, Andrew King, Grace Morris							
System Name									
Subsystem Name									
Component Name	Motor Casing								
Part # and Functions	Potential Failure Mode	Potential Effect(s) of Failure	Severity (S)	Potential Causes and Mechanisms of Failure	Occurrence (O)	Current Design Controls Test	Detection (D)	RPN	Recommended Action

Aluminum Casing	Casing rupture	Explosive decompression caused by rapid gas fluctuation	10	Voids in propellant grains, inconsistent aluminum rounds, incorrect milling	5	Grain analysis, Aluminum checks	4	200	Assign 1-2 team members to analyze each grain for voids, make sure during milling that aluminum is sound
Bulkheads	Bulkhead rupture	Rapid decompression which causes harm to the casing	4	Incorrectly machined parts, Faulty O-rings	3	Consistent dimension checks during machining	3	36	Make sure that consistent checks on the bulkhead dimensions are done, ensuring O-rings are consistent
0	Radial Bolt Failure	Destroys casing and bulkhead is no longer secure	5	Improper bolt selection	4	Load analysis of the bulkhead/ casing interaction	4	80	Ensure that the load analysis gives accurate results to ensure proper bolt selection
0	Pressure Transducer Thread Failure	Destroys bulkhead and pressure transducer	3	Incorrect thread tapping, Teflon tape use	4	Ensuring threads are fully cut	2	24	Make sure with machine shop managers that threads are adequately cut
Nozzle	Nozzle Improperly Secured	Nozzle unseats or falls out which would ruin compression and thrust	5	Incorrect seating, loose nozzle fit	3	Move the nozzle and look out for any slop or other signs of incorrect seating	1	15	Prepare an accurate CAD model to make sure nozzle seats correctly

Table 5 shows the potential failure modes of the motor casing. The major failure modes here that will be analyzed are the casing itself which can rupture as well as the bulkhead. These are two sections of the casing that are influenced by the internal pressure. According to Tripoli safety regulations, the max internal pressure of the casing should not exceed 1000 psi. This means that the casing must be designed around this factor. The bulkhead will have to have multiple O-rings to keep the pressure inside while also making sure the thread count is adequate for the pressure. There also must be consideration into the inside diameter to fit the liner. The casing is one of the most important parts of the project as it is the part that makes sure the pressure is sealed and only exhausts in the nozzle portion. The major calculations that must be done are axial and longitudinal stresses which can be used to find overall factor of safety.

Nozzle

Table 6: FMEA Nozzle - Sample

Product Name	NAU Rocket Club Capstone	Development Team: Shannon Comstock, Remington Dasher, Andrew King, Grace Morris				Page No of			
System Name	Carbon Rocket					FMEA Number			
Subsystem Name	Propulsion Subsystem					Date: 11/3/2023			
Component Name	75mm Rocket Nozzle								
Part # and Functions	Potential Failure Mode	Potential Effect(s) of Failure	Severity (S)	Potential Causes and Mechanisms of Failure	Occurance (O)	Current Design Controls Test	Detection (D)	RPN	Recommended Action
graphite insert for throat	Overpressurization of motor casing - Ductile Fracture	Explosion of motor casing, mounting points of nozzle are sheared off	10	Cross-sectional area of throat is too small	3	Measure precise throat diameter with micrometers	1	30	Pay special attention during design process, and ensure the insert has been machined to the correct diameter
	Throat area experiences extreme erosion	Decreased thrust performance	5	High temp. exhaust, abrasive granules present in exhaust gas	4	Simulations, and small scale experimental burn	3	60	Pick isotropic graphite, pick low-temperature propellant.
	Mechanical vailure via crack propogation	Fracturing of graphite insert, rapid decrease in thrust	8	Thermal expansion	2	Ensure proper fitting, and FEA simulations	3	48	Choose temperature resistant graphite, and consider adding insulation to reduce thermal expansion. Ensure the diameter is the same as the height of the insert
	Insert is ejected out of nozzle due to force of exhaust gas	The insert becomes a high-velocity projectile and the rest of the propulsion assembly may fail	10	If the step-down that holds the insert in place is too weak	1	Perform FEA on parts to ensure required strenth	2	20	include step-down geometry, precicely machine graphite for a press-fit, Heat up metal nozzle during press fit.
Converging-Diverging nozzle	Ductile failure of diverging section	Explosion of motor casing, rapid decrease in thrust	10	Nozzle diverges at too steep of an angle and the flow separation causes inintentional side-loading	2	CFD simulations in Ansys, and small scale test firings	2	40	Pay special attention during design process, and ensure the nozzle has been machined correctly
			9	The extreme temperatures weakens the design	3	Heat transfer simulations and hand calculations	2	54	Select nozzle material that has a high thermal conductivity and is resistant to melting
O-ring Seals	Force and/or temperature deformation	Increased likelihood of catastrophic motor failure, decrease in thrust performance, components that are not meant to experience extreme temperature will be affected by the escape of exhaust gas	10	Incorrectly sized O-rings; too small	2	ensure proper O-ring groove dimensions with calipers or micrometer	1	20	Choose temperature resistant o-rings or implement additional insulation
			10	Incorrect installation	3	Compare attached O-ring Diameter with inner diameter of casing	1	30	Ensure that the O-rings fit securely in the machined grooves in nozzle fitting
	Chemical and thermally induced corrosion	Chamber pressure will escape and have catastophic effects on motor and rocket parts	10	Manufacturer defect; did not cure properly in factory	1	Elastic strength tests	1	10	Implement quality assurance plan guring motor assembly

To ensure the safe operation of the rocket nozzle, several failure modes were considered. The first being the graphite insert at the throat of the nozzle. Typically, the throat of the nozzle experiences the most heat transfer, so the ablative properties of graphite will help keep the design within a safe operating temperature. It's possible that this component can crack due to thermal expansion, or if there is not enough material holding it in, can be ejected out of the motor casing. To prevent such issues, there will be thermal insulating paste between the insert and the phenolic piece to reduce heat transfer between the two. In addition, the design will include an adequately sized step holding it in place against the high-pressure exhaust gas.

Another part that is at risk of failure is the phenolic resin portion of the nozzle. This is also an ablative material but is less resilient than graphite. If this part is machined to be too thin, then hoop stresses may force it to come apart. If the diverging portion of the nozzle is too steep of an angle the flow separation of the supersonic gas may also force the assembly to come apart. These are issues that can be confronted and avoided through computational fluid dynamics software such as ANSYS Fluent.

The design will be tested with simulating and CFD software, as well as small-scale tests.

Equations and Solutions Required to Check Design Under Loading

Test Stand

The team has previously modeled the final design of the rocket motor test stand, depicted previously in figure 1. Components of this model were then analyzed in Ansys static structural analysis under loading. In Ansys, a crucial technique for physical modeling involves using contact elements to simulate interactions between different components in a mechanical system, especially beneficial for assemblies with moving parts or intricate geometries [3]. This approach ensures a realistic representation of contact dynamics under the loading which is expected from a M-Class motor.

When performing a static structural analysis using Ansys, various engineering assumptions can be used to simplify the analysis. One common assumption in Ansys static structural analysis is the linearity of material behavior under specified loading conditions, disregarding non-linear effects such as plastic deformation [4]. This simplifies the analysis, especially when applied loads fall within the material's linear elastic range. Additionally, boundary conditions are presumed to be accurately represented, and rigid body motions are typically constrained. The analysis also assumes isotropy in material properties unless specific anisotropic characteristics are defined. Thermal effects on the motor holder and bolts are considered negligible due to the exhaust direction of the motor away from this section of the test stand.

Within this analysis, the motor holder on the load cell is subjected a force of 287.8 lbf for each test, lasting around 4 seconds. The loading of the motor when ignited subjects the test stand to shear, and normal stresses, particularly within the bolts, brackets and motor holder. This evaluation aims to understand exactly what stresses these components will undergo during loading, ensuring that the loading remains well within acceptable stress limits.

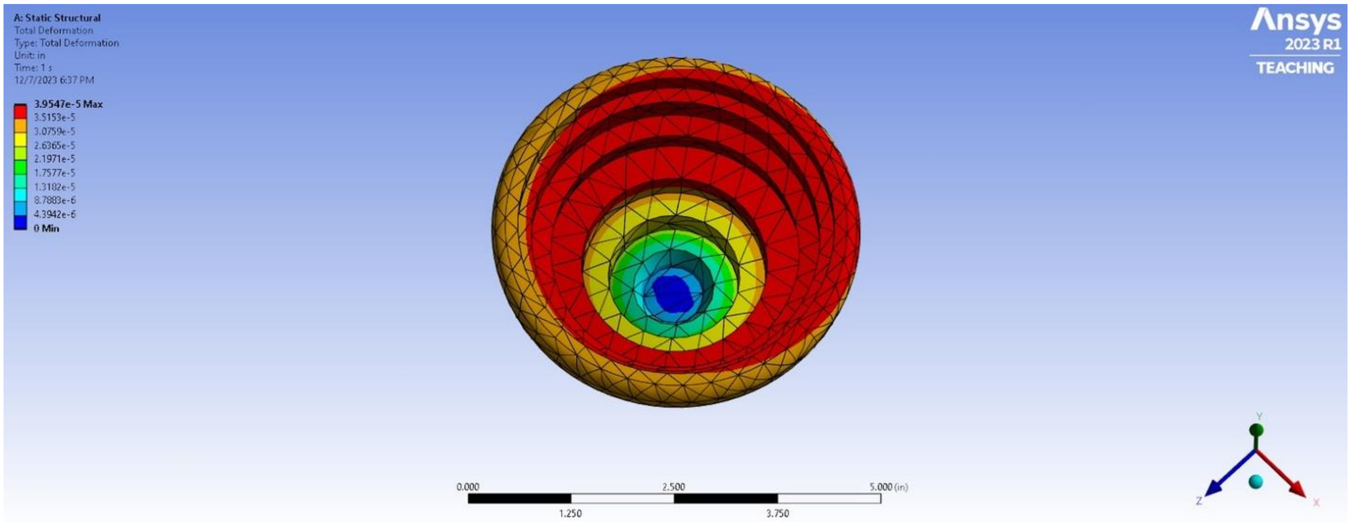


Figure 5. Test Stand Motor Holder Ansys Analysis

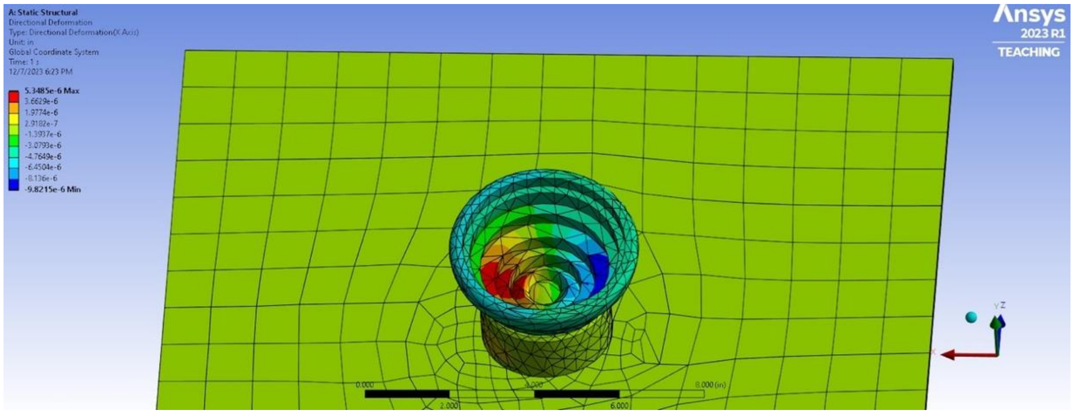


Figure 6. Test Stand Motor Holder Ansys Analysis

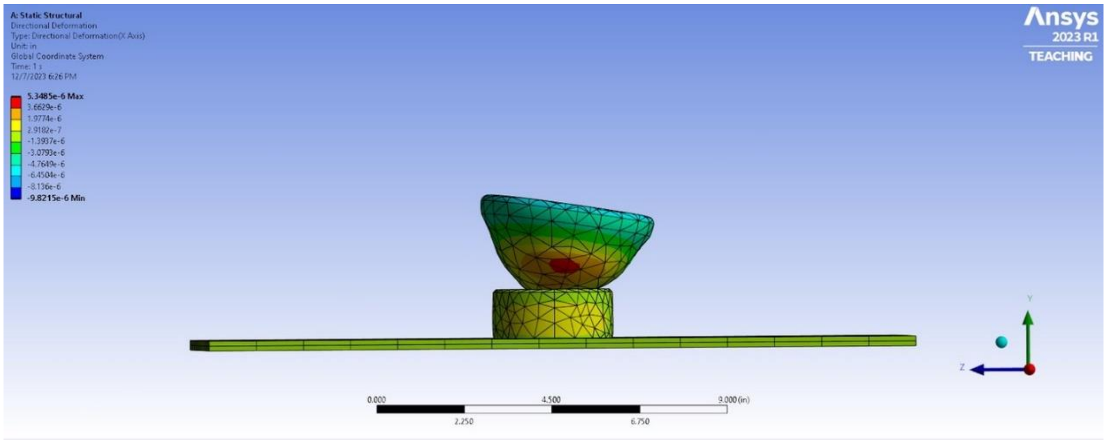


Figure 7. Test Stand Motor Holder Ansys Analysis

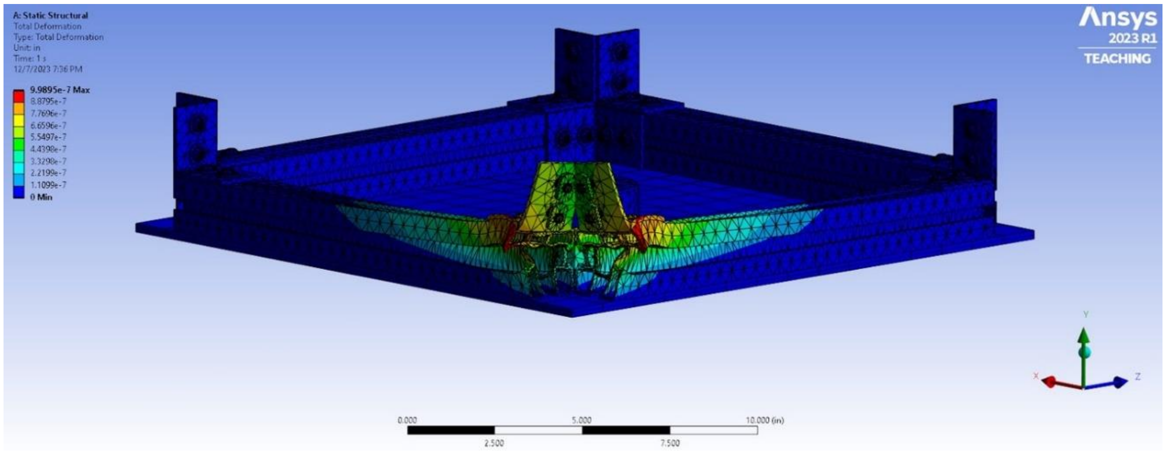


Figure 8. Test Stand Motor Holder Ansys Analysis

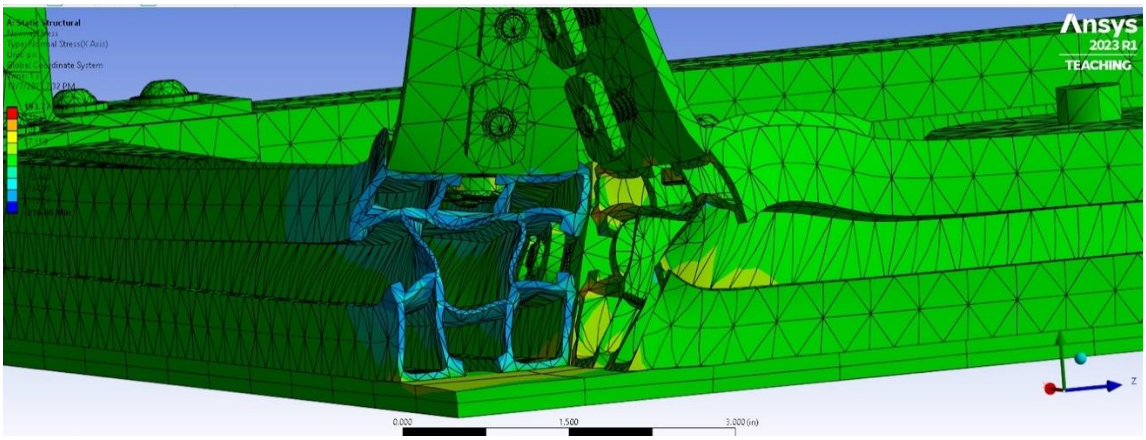


Figure 9. Test Stand Motor Holder Ansys Analysis

From this analysis, it is found that the stress concentrations on the base plate, extrusion bars, bolts and brackets is very small. Shown from the Ansys analysis in figure 8, the total deformation on the extrusion bar corner is $9.99x10^{-7}$ inches, or 0.0000001 inches. Due to this incredibly small deformation, is it determined that almost all force from the motor dissipates into the ground through the base plate before reaching the extrusion. Therefore, the extrusion, base plate, brackets and bolts are not subjected to significant enough loading for fatigue to affect their integrity.

In contrast, the motor holder is found to have a higher deformation shown in figure 5 of $3.95x10^{-5}$ inches or 0.0000395 inches. While this deformation is still very low, the criticality of this component functioning properly is much higher than other parts of the test stand. Therefore a factor of safety will be calculated for the design of the motor holder.

Rocket Motor

The geometry of the BATES propellant grain is depicted below in figure 9 in order to use for the calculations. Additionally, figure 10 illustrates the shear and bending moment diagrams employed in this shear stress analysis. From these models, an analysis of the shear forces on a motor grain is preformed using equations 1 through 3 shown below.

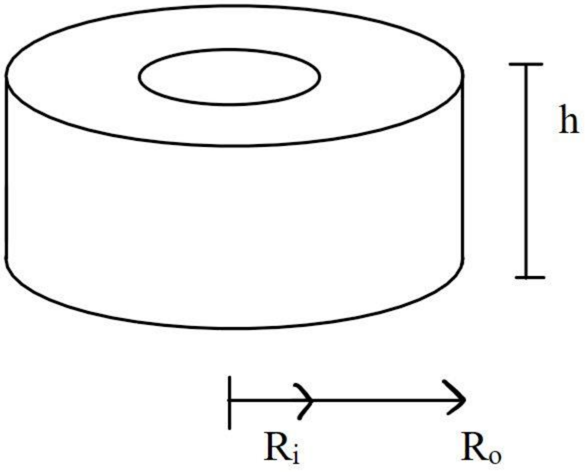


Figure 9: Propellant Grain

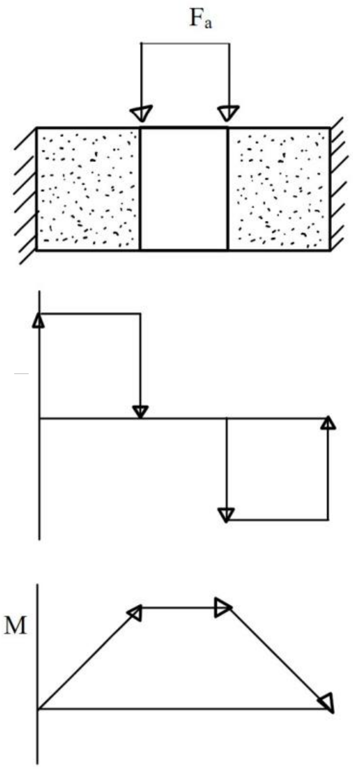


Figure 10: Shear and Bending Moment Diagram

T_m Equation 1.

$=$

$\cdot \frac{F}{}$

Equation 2.

A_n

$=$

$\cdot 2$

$\cdot \pi$

$\cdot r$

T_m Equation 3.

$=$

$\cdot T$

Table 7. Table of Known values for shear stress analysis

Variable	Value
r_{min}	7/16 in
h	4.5 in
	1.5 MPa [2]
F_{max}	280 lbf

By using the provided equations and the variable values shown in table 7, the team can assert with confidence that the propellant is resistant to shear failure. The maximum shear stress anticipated is 0.156 MPa, well below the propellant's capacity of 0.75 MPa, leading to a safety factor of 4.8. These computations were conducted based on the smallest grain size, considering the maximum force estimate. This choice was made because the smallest grain exhibits the smallest area, resulting in the highest shear stress.

Motor Casing

The motor casing has many regulations and guidelines associated with it. According to the Tripoli Rocketry Safety Standards, the casing must be made from a nonferrous material that is ductile. The common materials that are used is aluminum alloys due to their high specific strength. These safety rules are in place due to the extreme pressures these casings experience. Ferrous materials can cause sparks which is not ideal for a highly combustible material that's being encased. Brittle materials fragment when they fail which means that the high pressures can send sharp fragments of metal everywhere which is very unsafe. To design an appropriate casing, factors of safety have been determined already. The FOS that must be used is set at 1.5 [1]. This factor of safety will be used to calculate maximum pressure for the casing. After this is completed, the maximum allowable pressure given by Tripoli (1000 psi) will be used to find the adjusted factor of safety.

$$\frac{r}{t} \geq 10 \rightarrow \frac{(3-(2\cdot 0.125))}{0.125} = 11 \text{ (Thick-Walled Assumption)}$$

Hoop Stress: $\sigma_t = \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 + \frac{r_o^2}{r_i^2}\right)$

Rearranged for internal pressure: $p_i = \frac{\sigma_t}{\left(1+\frac{r_o^2}{r_i^2}\right)} \frac{(r_o^2 - r_i^2)}{r_i^2}$

After solving for max tangential pressure in the casing, we get **2314.47 psi**

The adjusted Factor of Safety, we get **2.314**

Longitudinal Stress: $\sigma_l = \frac{p_i r_i^2}{r_o^2 - r_i^2}$

Rearranged for internal pressure: $\frac{\sigma_l (r_o^2 - r_i^2)}{r_i^2} = p_i$

After solving for max longitudinal pressure, we get **4848.48 psi**

The adjusted Factor of Safety, we get **4.848**

Nozzle

To better understand the conditions of the exhaust gas within the nozzle, a few MATLAB programs were created based on the formulas found in Sutton & Biblarz Rocket Propulsion Elements, 9th Edition [5]. The most useful of the programs is the one based on example 3-2 of Rocket Propulsion Elements. The first values calculated with the program is ideal thrust, F, and ideal specific impulse. Next, cross-sectional area, A, Mach number, M0, are plotted with respect to pressure along the length of the nozzle.

The first equation solves for effective exhaust velocity (equation 4). This equation is true under the assumption that the flow is adiabatic and isentropic. For the sake of creating a perfectly expanded nozzle, exit pressure, p2, is equal to ambient pressure, p3.

$$v_2 = \sqrt{\frac{2k}{k-1} R T_1 \left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{(k-1)}{k}}\right]}$$

Equation 4.

In the equation above, the heat capacity ratio, k, is required. It is a difficult value to get by hand, so this value was obtained with Burnsim and NASA’s Chemical Equilibrium Applications. Setting v2 equal to effective exhaust velocity, c, the following equations are solved:

$$I_s = \frac{c}{g_0} \qquad \text{Equation 5.}$$

$$F = \dot{m}c \qquad \text{Equation 6.}$$

Next, plots are generated with the equations shown below. P_y represents variable pressure, and that is what the plots are based upon. Values found with the equations include specific volume, temperature, velocity, cross-sectional area, and Mach number.

$$V_1 = \frac{RT_1}{p_1} \qquad \text{Equation 7.}$$

$$V_y = V_1\left(\frac{p_1}{p_y}\right)^{(1-k)} \qquad \text{Equation 8.}$$

$$T_y = T_1\left(\frac{p_y}{p_1}\right)^{\frac{(k-1)}{k}} \qquad \text{Equation 9.}$$

$$A_y = \dot{m}\frac{V_y}{v_y} \qquad \text{Equation 10.}$$

$$M_y = \frac{V_y}{\sqrt{kRT_y}} \qquad \text{Equation 11.}$$

Factor of Safety Table

Next, create a table that summarizes the minimum FoS (Factors of Safety). Highlight any areas of concern in your design (FoS<1.0) and show images of parts for clarity.

(Everyone) (Shannon helps Remy does motor casing and strand burner)

In ensuring that each component within the project is designed to avoid potential failures, a calculated factor of safety is integrated throughout the design process. Table **, presented below, outlines the factors of safety utilized in the design process. Most of these safety factors have been individually calculated based on parameters for the components, while others have been determined by referencing rocketry standard resources. The process of determining each of these safety factors will be presented in the subsequent discussion.

Sub-system	Component Name	Load Case	Material	Minimum Factor of Safety
Test Stand				
	Motor Holder	280 lbf applied directly to the inside of the motor holder	Aluminum	3.37
Rocket Motor				
	Propellant Grain	280 lbf applied to the inside of a 38 mm propellant grain	HTPB R45	4.8
Motor Casing				
	Casing Tube	Max internal pressure of 1000 psi	Aluminum	2.314
	Bulkhead	Max internal pressure of 1000 psi	Aluminum	4.848
Nozzle				
	Ablative Phenolic	Maximum shear stress of 202.11psi	Linen Impregnated Phenolic Resin	66.762
	Internal Circlip	Maximum thrust load of 634.63psi	Black Phosphate 1060-1090 Spring Steel	60.508

Test Stand

In order to determine the appropriate factor of safety for the motor holder, the material needs to be incorporated as well as the forces which the motor holder will experience. Since the motor holder will be made from aluminum, the allowable stress for aluminum needs to be applied into the factor of safety equation $F.O.S. = \frac{ultimate\ stress}{allowable\ stress}$ where the allowable stress of aluminum is approximately 276 MPa [5] and the ultimate stress is 310 MPa. Therefore the minimum factor of safety for the motor holder is 1.12. In order to ensure safety for the integrity of the motor holder, the factor of safety should be tripled to be 3.37.

Motor Casing

The initial calculations were done with a strength factor of safety of 1.5. After this is applied to the yield strength of the aluminum, the hoop and axial stresses can be calculated. This will give us a value for what internal pressure is allowed with that factor of safety. Then, knowing the maximum internal pressure, this can be used to find the adjusted factor of safety. Knowing this value will allow us to determine exactly what thickness we can use for our casing to have an appropriate casing design. It is extremely important that the casing is designed appropriately to ensure that casing rupture does not occur. This is even more important while inside the rocket so it does not destroy the rocket body. Knowing the calculated values that were preformed are imperative to the success of the project.

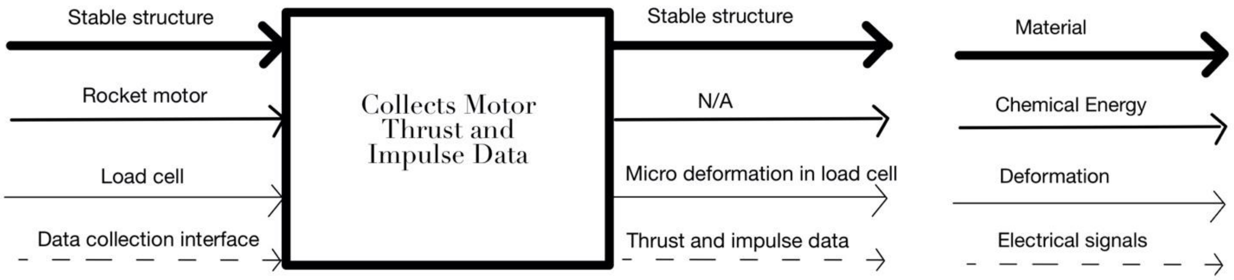
Nozzle

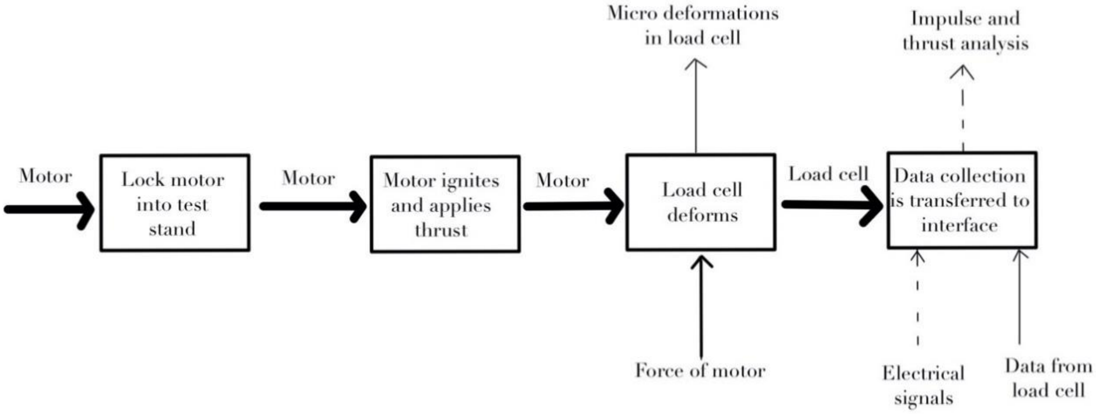
To ensure the safety of the nozzle assembly, the team simulated thrust force experienced by the nozzle at the maximum allowable chamber pressure of 1000psi. According to the Matlab program discussed in the engineering calculations section above, the force exhibited by the nozzle will be 634.63lbf in a worst-case scenario. The Circlip holding the nozzle in place will have to resist this thrust. The clip the team has chosen is heavy duty and can withstand a thrust load of 38,400lbs. This is way larger than our expected force, so the team should investigate buying a smaller circlip. This decision will save weight while keeping the safety factor of the circlip at a safe number.

As for the shear force being experienced by the phenolic resin, this was calculated to be 1.393e+6 Pa. The shear stress of the linen phenolic is 93e+6 MPa. This is quite a large safety factor. This opens the possibility to save weight in the design by taking out material, but for the now the team will keep the design as-is for peace of mind.

Flow Charts and other Diagrams (20pts)

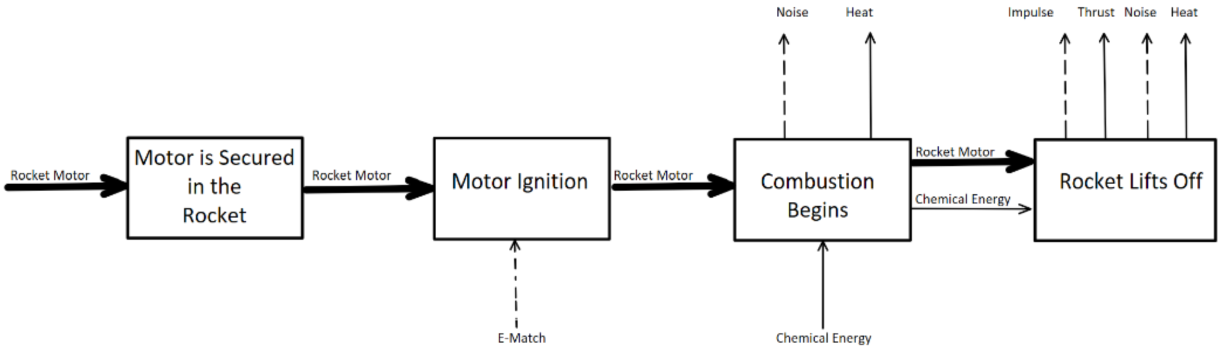
(Grace and Shannon)





The black box model and the functional flow model for the rocket propulsion system are shown below. The black box model helps the team to identify the key inputs and outputs in this sub-system. The primary input aside from the actual rocket motor is chemical energy. The primary outputs that should be seen at the launch are thrust, impulse, heat, and noise.

The functional flow model shows the energy and materials as it moves thought the system. This allowed the team to ensure that all necessary steps will be met through the team's chosen design. The primary step is to make sure that the combustion begins smoothly. If there is a problem with combustion, then the rocket will fail to meet the requirements set by the client. The second most important step is to ensure that the motor ignited properly. If there is a problem in this step, then the flow will stop, and the rocket will fail.



Both of these models allowed the team to identify key points for possible frailer in the motor. Additionally, this guides the way the team will measure the progress of the design.

Moving Forward (Grace and Shannon)

The next steps for this project are mainly focused on the engineering calculations for the motor casing. In order to complete these calculations, a code the team has made in MATLAB will be used to determine the casing wall thickness as well as the hoop stress. A few iterations of this calculation have been made, however the values provided from the code need to be tested in software such as Ansys to ensure that the pressure forces applied to the motor casing will not fatigue this component.

Aside from the motor casing, the steps moving forward do not require more engineering calculations as all designs and components have been analyzed through modeling software's. Therefore, moving forward the team will focus on assembling, manufacturing and testing the designs which have been formulated in order to have the final rocket ready to launch in March of 2024.

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Appendix A

Tripoli Rocketry Safety Code	
Code #	Description
1-2.1	This code establishes guidelines for the reasonably safe operation of rockets at Tripoli Launches.
3-1.2	A Tripoli member shall fly a rocket only in compliance with the regulations controlling the airspace in the country where the launch is held. This applies to other applicable federal, state, and local laws, rules, regulations, statutes, and ordinances. It’s the responsibility of every Tripoli member to understand the local regulations.
6-7	Range Personnel may access any portion of the range as directed by the Launch Director or RSO.
7-1.4	The thrust-to-weight ratio of a rocket typically should be at least 5:1. However, the RSO may approve a thrust-to-weight ratio as low as 3:1 ratio.
7-2	The rocket must remain pointed in a safe direction (away from all people) while installing the igniter and using the igniter then after.
7-5	When needed, blast deflectors must be used to prevent damage or reduce the risk of fire.
7-6	Blast deflectors shall be oriented so that any ejected motor parts shall not endanger people.
7-8	The motor igniter shall not be connected to the launch system until all other flight electronics are active.
7-10	Range activities shall cease whenever a thunderstorm has been detected within ten miles (16 kilometers) of the launch site.
7-12	Alcohol or drug use which might affect judgement or response to hazardous situations is prohibited for all persons during Tripoli Launch operations.
9-1	Launch control operator shall not launch a rocket that has not been announced.
9-2	The announcement shall include a countdown of at least 5 seconds.
9-3	No rockets shall launch when the sustained surface winds exceed 20 MPH (32 KPH)
9-4	Rockets shall not be launched in any way that could interfere with aircraft operations.
9-5	No rocket shall be intentionally launched through any altitude with greater than 50% cloud coverage.
9-6	No rocket shall be intentionally launched into or through a cloud.
10-10.1	Metallic cases shall be made of non-ferrous ductile metals such as 6061 aluminum alloy.
10-10.3	Forward closures shall not be made of ferrous materials.
10-10.4	Minor components such as snap rings, nozzle washers, rear closures, and seal disks may be made of ferrous materials.
13-6	A high power rocket must be launched no more than 20 degrees from vertical.

13-7	Igniters shall not be installed in high power rocket motors except at the pad or at special preparation area away from all uninvolved people.
13-10	Where possible, igniters must be removed and all sources of ignition must be disarmed before lowering a rocket from launch position.
13-11	A mercury switch or roller switch shall not trigger the ignition of a rocket motor.
13.17	Safe Distance Table (See on Tripoli website)