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To: Dr. David Willy

From: 2 – Haptic Robot

Date: 9/8/2023

Re: Engineering Calculations Summary

Introduction

This memo is being written to elaborate on the work that has been completed on the Haptic Robot project for our client, Dr. Reza Razavian. Below, you will find a design summary that goes into depth about the current design the team has in place, as well as the customer and engineering requirements that were the foundation for the design. The memo also reviews the standards, codes, and regulations that were applied to the design just before the equations and solutions that were used for the calculations of the robot. Continuing on in the memo are the charts and diagrams used during the design process of the robot as well as the work that will be done in the future to ensure the team delivers a quality product on time to the client.

Top Level Design Summary

Haptic Technology has proven to be very effective in a number of industries including the medical field. The team was tasked with creating a haptic robot that has 3-Dimensional movement and will be used for rehabilitation. Patients who have suffered from a stroke or have cerebral palsy can benefit from using this robot during their rehabilitation process and could possibly reduce the time they spend recovering. In order to design and build this robot the team has come up with a design that moves in three degrees of freedom and produces 20 Newtons of force. Below is the detailed design of the robot and all the subsystems that make up the robot.

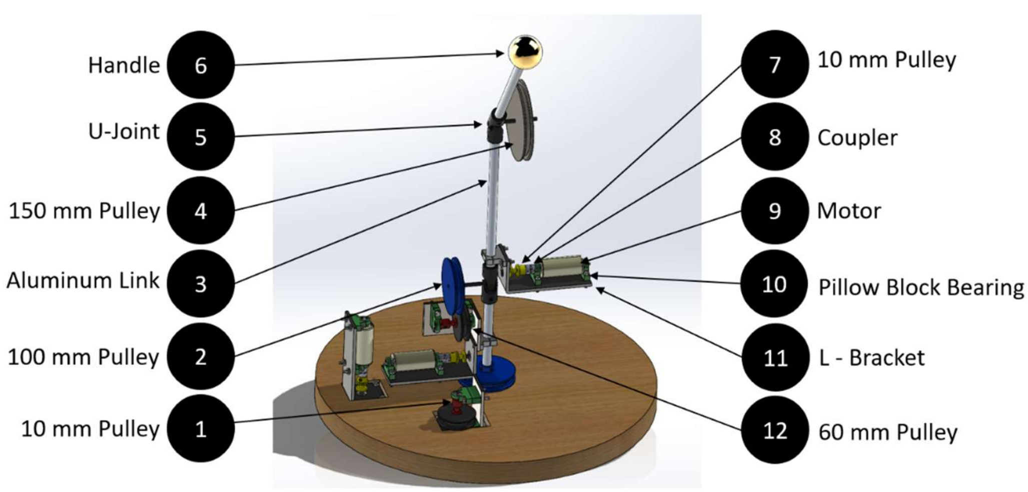


***Figure 1**: CAD Drawing Labeled with Balloons*

Figure 1 above shows twelve different balloons that point to a number of different parts within the robot. To help breakdown the design of the robot, the team has decided on three different subsystems that we felt were important. The first subsystem that will be reviewed is the Motor Mounts. In Figure xxx above there are three motor mount assemblies. Balloons 7 through 11 represent one motor mount. The first part within the assembly is the yellow pulley which is labeled balloon 7. This pulley is one of the ten pulleys in the system, which will be gone over in greater detail in the next paragraph. Balloon 8 labels a coupler to attach the shaft and the motor of the robot, which is labeled by balloon 9. Finally, balloons 10 and 11 show the pillow block bearing that is used to attach the motor to the L bracket. This assembly is recreated two other times and placed at different parts of the robot to power the design.

As previously stated, there are a total of ten pulleys within the team's design. The pulleys are color coordinated in figure xxx above, but to go into more depth there is an arrow pointing to each of the different sized pulleys within the system. The first pulley which was also covered in the paragraph above is the yellow pulley, or balloon 7. This pulley is 10 mm in diameter and is duplicated throughout the design to drive the pulleys labeled by balloons 4 and 12. The light gray pulley is 150 mm in diameter and the black pulley is 60 mm in diameter. Balloons 1 and 2 points to a red and a blue pulley. The blue pulley is 100 mm in diameter and the red pulley is also 10 mm in diameter. This helps the viewer distinguish which pulleys are a part of which systems. You will find that this design has a lot of repetition, the pulley system uses the same size driver pulley for each driven pulley, no matter the size.

Finally, to finish the design explanation, balloons 3 and 5 point to the links and joints of the robot. The current joints used in the design are universal joints and there are two of them in the design. In the future work section of this memo, the team will go into greater detail about some of the other joints that are being considered for the design. The links of the robot will be made from aluminum. The reason for this is because aluminum is a very lightweight material, which allows the team to meet some of the customer requirements, all while keeping the structural integrity of the robot and staying under budget. Some of the other key components of the design are the handle of the robot, which is labeled balloon 6, and the base of the robot which is made of wood.

Customer & Engineering Requirements

Customer Requirements

Project requirements are set by Dr. Reza Razavian to develop a lightweight robotic arm with three degrees of freedom to be used for patient rehabilitation with motor skill impairments. For the project, the team has taken into consideration customer requirements and the engineering requirements of the project. Some of the customer requirements have been translated into engineering requirements so that the team can set specific goals the final design must meet. The specific customer requirements are listed below and are rated based on what the team and client agree is most important, a rating of 5, to least important, a rating of 1. The engineering requirements are listed further down and are equally important. They have specific goals to be met that are calculable for efficient evaluation of the design

1. **Lightweight- 5** The robot must be relatively lightweight so that the arm can move quickly with ease. This is more to decrease the moment of inertia from any movement. Links/arms being lightweight will help maintain the structural integrity of the robot and enable it to move fast with ease.

2. **Affordable- 3** The budget is given, it is $5,000.00 USD and has little room for flexibility. The client has understood the budget might need to increase depending on part costs that are approved of. The team is now tasked with trying to stay under the budget if possible. Based on current designs of the robot, the team is underbudget and will most likely not need more than $2,000.00 USD.

3. **User Friendly- 3** For the robot to be effective it must be easy to use. This entails safety as well as simplicity for the user to gain rehabilitative benefit. The robot will use minimum parts/outer casing to reduce pinch points and other potential harm that could be caused by the robot moving. The handle will be a sphere or a different ergonomic design, so the user is comfortable.

4. **Stiff- 5** Depending on how the robot links are designed, the material needs to be stiff-standing. It must not move easily from being pushed or wobbled. Between the material properties and the link connections, the robot must be locked in place unless it is intentionally moving. Notably, motors must have a higher torque rather than only high RPM for this to be achieved.

5. **Accurate- 4** Motors need to be able to have a program work in tandem with their controllers so that they can move the handle to any point in space required. 3 degrees of freedom are required for this to be achieved, which means there must be a minimum of 3 motors. The motors and controllers must be high quality enough to have superior accuracy.

6. **Flawless Motion- 5** Similar to stiffness depending on the design on the links, flawless motion requires unique design so that there is minimalized friction within the mechanical system. Gears can cause friction, depending on type, quality, and size. Looking into other subsystems to turn links in a degree of motion may prove useful to meet this requirement. Machining parts so that they do not overlap is also important to lower friction, as well as fastening pieces tight.

7. **No Backlash- 5** Gears are known for causing backlash since making them mesh perfectly is almost impossible. Not only do gears wear down over time, but they also need to be lined up in points in 3D space with each other to maximize efficiency. It may prove difficult to purchase gears that increase torque and do not cause backlash, even over time.

8. **No Friction- 4** To have flawless motion, there needs to be little to no friction, so gears are yet again difficult to use and satisfy this requirement. To limit friction means to maximize the power output and speed response with the robot. If the arm is moving from rest, having no friction between parts will also make it more user friendly so the force required to move the robot is more accurate to the desired setting.

9. **Produce Force- 5** The arm of the robot will hold the handle at the end and needs to be able to replicate a physical therapy exercise. This is the haptic rendering features of the robot, where it can respond to force input and give a resistance output. Electrical energy needs to be turned into mechanical energy at the motors, then transmitted through the arms to the user, so the energy the user experiences needs to be reasonable force.

Engineering Requirements

1. **Decrease Weight**

Target goal: <50lbs The client requires the finalized product to be lightweight, at least the robotic part. The base of the robot will hold the most weight so that the moment of inertia will not cause the robot to tip over and fall. This is why there is a weight limit of 50 lbs.; so that the robot is light enough to relocate if needed, but heavy enough to resist tipping while the robot arm is moving.

2. **Tolerance (Reliability)**

Target goal: <1% of any requirement This is a general tolerance for any measurement required by the client or general engineering requirements. This will allow the team to easily calculate and predict how the robot's components will react to loads. By using MATLAB Simulink, the team will organize all calculations through a program which will provide useful information for the team to consider. These numbers tell the team if building the robot with specific materials will work or not.

3. **Material Strength (Durability)**

Target goal: >2 GPa Material strength is important for the robot to last rough usage over time. A higher material strength provides resilience against such usage permanently damaging the robot, so tough motors as well as strong housing for the robot's components are important.

4. **Force**

Target goal: >20 N The robotic arm must produce a maximum force around 20 N to have capabilities of therapeutic value. With an average finger push being around a newton, our client has instructed us that 20 newtons will be enough for physical therapy purposes. This requirement is a minimum, so it is possible/ likely we can design for the robot to produce more force.

5. **Reduce Friction**

Target goal: <1 N The robot must be able to move freely without friction to prevent wasted energy and speed reduction. This will be accomplished by not using gear systems (unless necessary) as well as having the fewest moving parts possible. If a cable design is used, friction is necessary but can be managed to create more torque.

6. **Speed**

Target goal: >1m/s Speed and accuracy go hand in hand between customer and engineering requirements. The team must design a robot with an arm that can move at minimum 1 m/s speed to an accurate position in 3D space. This requires a superior controller for the motors as well as a coordinated program to maximize robot usage.

7. **Electrical Power**

Target goal: >100W With The requirement of having 3 degrees of freedom, our design will use 3 different motors. This will require a lot of power, along with the controllers for each motor. With this engineering requirement, the team is ensuring use of a mechanical system that is powerful enough to meet other requirements such as the output force of 20 N.

QFD

Below is the teams QFD broken into smaller parts; the entire QFD can be found as *Appendix A* in the appendices section. The first section is a review of the previously stated customer requirements, and how they are weighted in relation to the amount of importance they have in the project. Some of the most important requirements include: a lightweight design, flawless motion, and produce force. The user experience and affordability of the design are still important, just not as high on the list as the previously mentioned requirements.

	Customer Needs	Customer Weights
1	Lightweight	5
2	Affordable	3
3	User Friendly	3
4	Stiff	5
5	Accurate	4
6	Flawless Motion	5
7	No Backlash	5
8	No Friction	4
9	Produce Force	5

Figure 2*: Weights of Customer Needs*

Figure 3 shows the technical requirements for the project and is the foundation for the rest of the QFD. In *Figure 4*, you can find a grading scale that compares the customer and technical requirements to each other and grades them based on the correlation they have to one another. There are four different grades given in the table: 1 is little correlation, 3 in some correlation, 9 is high correlation, and blank means there is no correlation between the two.

Technical Requirements						
Decrease Weight	Tolerance	Material Strength	Force	Reduce Friction	Speed	Electrical Power

Figure 3*: Technical Requirements*

		Technical Requirements						
Customer Needs	Customer Weights	Decrease Weight	Tolerance	Material Strength	Force	Reduce Friction	Speed	Electrical Power
Lightweight	5	9	3	9		1	9	1
Affordable	3			1				1
User Friendly	3	3				3	9	
Stiff	5		3	9	3	1	3	
Accurate	4	3	9			3	3	9
Flawless Motion	5	3			1	9	9	3
No Backlash	5	9	9	3	3	3	9	1
No Friction	4	9			9	9	9	
Produce Force	5	3		9	9	1	9	9

Figure 4*: Grading of Technical Requirements & Customer Needs*

After grading the technical requirements and customer needs, the last step of the process is to calculate the absolute and relative technical importance of each technical requirement. This shows which technical requirements the team needs to focus the most on throughout the design and testing process. In *Figure 5* below, you will find the relative and technical importance values. This figure also shows the target values for each technical requirement as well as the units for each. The first target value is 50lbs which falls under the decrease weight category. A very important customer need is a lightweight design, and after speaking with our client, the consensus was a goal weight of 50lbs with hopes to have an even lighter design. This allows the robot to move quicker as well as be easily transported if needed. To reduce weight while keeping a material strength of 2 GPa, the team feels carbon fiber would be the best option to use as the material for the robot. The robot must produce some sort of force to be used for rehabilitation therefore the client gave the team a target value of 20 N of force. The client also wants the user to be able to do one full arm movement across the body in approximately 1 m/s therefore in order to do so, the robot must have very little friction leaving the groups target value for friction 1 N and speed 1 m/s. Overall, the team would like to have a tolerance value no greater than 1% which allows for a high quality product and avoids any damage to the robot. Finally, the power output must be around 100 watts in order for the robot to perform up to the client's standards.

Technical Requirement Units	lbs	%	Gpa	Newtons	Newtons	m/s	Watts
Technical Requirement Targets	50	1	2	20	1	1	100
Absolute Technical Importance	162	93	120	110	129	261	107
Relative Technical Importance	2	7	4	5	3	1	6

***Figure 5**: Technical Requirement Units*

Summary of Standards, Codes, and Regulations

The Haptic Robot capstone team referred to the National Society of Professional Engineers (NSPE) Code of Ethics as the main guide for design production to ensure the project is within the limits of the established codes of ethics and to provide a safe, trustworthy, and appropriate design for the client and the public. An important fundamental canon that is considered to be the first code in the team's design according to the NSPE is to "Act for each employee or client as faithful agents or trustees" [1]. In every step that the team has taken in the process of the design has been thoroughly analyzed and presented to the client to ensure that the code of ethics mentioned is effective during the design process. The other important code of ethics that is effective in the team's design process is also from the NSPE Professional Obligations that states, "Engineers shall be guided in all their relations by the highest standards of honesty and integrity" [1] which applies to the team's efforts to have an original design plan that is based on team members knowledge and client and engineering requirements. In addition, using any external sources for guidance is always referenced and mentioned appropriately in relation to the team's project.

***Table 1**: Standards of Practice that are Applicable to this Project*

Standard Number or Code	Title of Standard	How it applies to Project
NSPE (I-4)	Fundamental Canons	Ensures that any work done on the project is presented and approved by the client.
NSPE (III-1)	Professional Obligations	Used as a guide to provide quality work with original design ideas that are based on honest efforts, integrity, and previous knowledge.

Summary of Equations and Solutions

The mechanism of the 3-degree robot that the team is designing requires certain analyses to ensure engineering requirements are met such as torque, gear ratio, stiffness, and deflection rates in each of the three different links of the robot. The load applied to the handle of the robot is a maximum of 20 N which needs to be supported by the motors to provide enough torque countering the load applied and ensuring appropriate resistance. The general team engineering analyses of the robot were divided into the main systems that are required to ensure robot functionality which are the pulley systems, first link, second link, and third link respectively. The analyses focused on the weak points of each system and the calculations were made to conclude that each system will function according to engineering and client requirements.

<u>Pulley System</u>

Speed and Rotation - Logan Schubert

Based on the force analysis done by other team members we found that the speed of the motor would be 2000rpm. The base pulley and link 1 pulley have the exact same set up which make the calculations a little easier to solve for. For those pulleys they have two sets for four reels total. As for the second link pulley system there is only one set of two reels making solving for the speed and ratios simple. The given values for the reel diameter and speeds are in *Appendix B*.

There are two sets of pulleys in these systems. Starting with the driver pulley at 10mm in diameter moving a 2000rpm is linked to the driven pulley with a diameter of 60mm. On the same shaft as the 60mm pulley is the next set with a 10mm driver linked with a large 100mm driven pulley. Having the initial driver speed of the motor and the diameters of all the pulleys we can do a few calculations to find the speeds and ratio.

Based on the calculations done in MATLAB I found that the final speed of pulley system would be 33.33rpm with a total ratio of 1:60 seen in *Appendix C*. This is a sound calculation based on the weight and torque needed towards the bottom of the robot. A gear reduction was added based on our previous calculations because there would not be enough torque to move the base or even the second link. The first pulley set has a ratio of 1:6 while the second set has a ratio of 1:10.

For link 2, there is only one set of pulleys in this system. Starting with the driver pulley at 10mm in diameter running at a speed of 2000rpm. The driven pulley that is at the top joint has a diameter of 150mm being the largest of all the pulleys. Since I was given the initial speed and the pulley diameters, I can determine the speed and ratio of the pulleys on the top of the robot.

Based on my calculations from MATLAB for the final pulley system I found that the total ratio is 1:15 with the running speed of the driven gear being 133.33rpm seen in *Appendix D*. The reason for making the top pulley so large was so that when the top link is in motion there would be a high enough ratio for there to be enough torque for the pulley. The top link will have the least amount of weight but will also be a crucial part of the user interaction with the robot. Overall, the pulley speeds meet the requirements given by the client based on the force analysis.

Tension in Pulleys – Christopher Hernandez

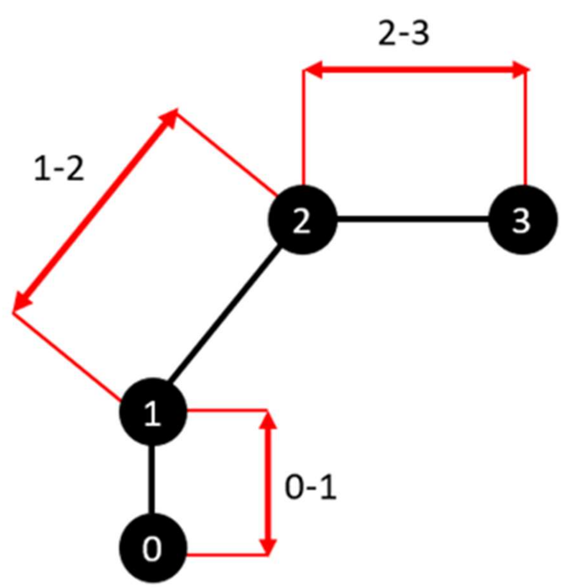


Figure 6: Basic Design of Robot Showing Links and Joints

Prior to doing individual analysis on the system, the team did a force analysis of the entire robot. From the Force analysis, the team was able to find the moment applied at each joint which would also be the moment applied to each pulley. Based off the calculations in the Force analysis of the robot, the team discovered the motors being used do not supply enough torque to power the robot, leading to a larger reduction needed in the system. The original reduction for all of the pulleys in the system was about a 1 to 20 reduction. In order to power the robot, the team needs to increase two of the reductions to a 1 to 60 reduction. With all of the smaller pulleys being 10mm in diameter, a 1 to 60 reduction would not work with the design of the robot therefore the team needs to add two more sets of pulleys to the system. In *Figure 6* below, you will find three different sets of pulley systems and a total of ten pulleys for the robot. This section of the memo will go over the moment in each pulley system based off the force analysis, as well as the tension in each of the cables of the pulleys.

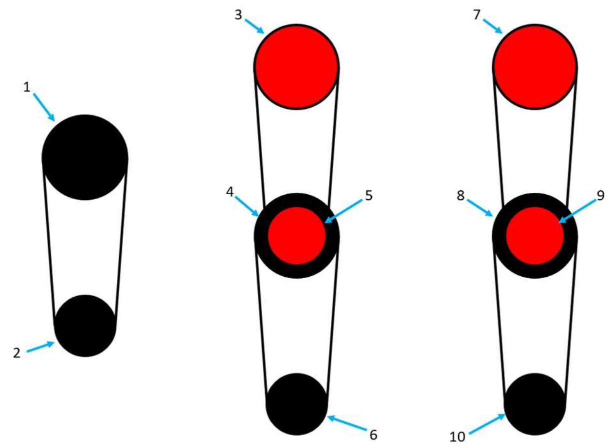


***Figure** 7:* Pulley Systems in Robot

Link 1 Pulley

In *Figure 7* above, you will find the pulley system for the link between joints two and three as shown in *Figure 6* above. From the force analysis calculated by the team, pulley number one has a moment of 13.2 Nm. To find the amount of tension needed in each cable, you must use equations 1 and 2 below, where T_1 and T_2 are the tensions in the cable, C is the tension the user will set the cable to before using, M is the moment found in the force analysis, and r is the radius of the pulley being analyzed.

$$T_1 + T_2 = C \tag{1}$$

$$r(T_1 - T_2) = M \tag{2}$$

In order to find the tensions in the pulley cables, you must first find the preset tension, or the tension that is set prior to the use of the robot. In order to find the preset tension of the cables, you must use equation 3, where r is the radius of the pulley being analyzed, C is the pretension value, and M is the moment or torque being applied to the robot.

$$r \times C = M \tag{3}$$

By plugging in a radius of .05m, and a moment of 20Nm into equation 3 above, you are able to find the preset tension value for this system is 400 Newtons. You are then able to insert 400N as the value for C and 13.2 Nm as the value for M in Equations 1 and 2. This leads to two equations with two unknowns, and you are now able to solve the tensions in the cables of the first pulley system. After doing basic calculations, the Tensions in the cable under 13.2 Nm of torque are 332N and 68N as seen in *Appendix E* below.

As stated earlier in the memo, the pulleys need to have a reduction ratio of 1 to 60. In order to fit the pulleys within the current design of the robot, the group must add another pulley system to the first link. The initial pulley diameters are 100mm for the larger pulley and 10mm for the smaller. The diameters for the second pulley system attached to this link are 60mm for the larger pulley and 10mm for the smaller. The process for finding the preset tension in the cables as well as the tensions of the cables under a load is the same as before. To find the amount of preset tension in the cable, you would divide the initial moment applied by 10 since the system has already gone through a one to ten reduction. The values applied to equation 3 are 2Nm for M, and .06m for r, which gives a preset tension value of 66.667N. Plugging .06m for r, 1.32Nm for M, 66.667N for C, and solving for T_1 and T_2 you will find the tensions in the cable for the second pulley of the first link are 55.333N and 11.333N. (*Appendix F)*

Link 2 Pulley

The calculations for the rest of the pulleys are repetitive of the first set of calculations. To begin the analysis of the second link pulley systems (*Appendices G & H*) you plug in .05m for r, and 20Nm for M to get the same value of 400N for the preset tension in the first pulley system for this link. Moving forward to equations 1 and 2, using .05m for the radius, 400N for the preset tension value, and changing the value for M to 13.0019Nm which was found during the force analysis, you will find the T_1 and T_2 values for the first pulley system in this link are 330.0190N and 69.9810N.

Following the same pattern as the previous link with a second pulley system attached to this link, and dividing the input values by 10, the values plugged into equation 3 are: 2Nm for M and .06m for r, you will find the preset tension value is again 66.667N. Moving into equations 1 and 2, with .06m as the radius, 1.30019Nm as the M value, and 66.667N as the C value, the two tensions in the cable are 55.0032N and 11.6635N.

Link 3 Pulley

The third link differs from the first two because it only has one pulley system attached to it instead of three. While the overall design is different, the calculations are the same as seen in *Appendix I*. Using equation 3, with .075 as the radius and 20Nm and the moment, the preset tension value for the cables in this system is 266.667N. Using this value in equations 1, along with a .075m radius, and a 6.1047Nm moment, the values for T_1 and T_2 are 174.0313N and 92.6353N. This system did not need two sets of pulleys because the moment in the pulleys is significantly less therefore the team is able to fit the pulleys within the current size of the design.

Links & Joints

The First Link

To give some background to our design and materials used for the project all the links are made from the same material. We used aluminum pipes with an outer diameter of .75 inches with an inner diameter of .652 inches. We chose this material because it is lightweight and strong enough to withstand the forces we would be applying to the robot. This materials over yield strength is 35000 psi [5] for the pipe which is more than enough for our application.

Together as a team we had performed some initial calculations to get us all started on the same foot, so we could later go on to complete our assigned analysis for our specific parts. In this team analysis we had to start by calculating the moments at the joints in a static position which are transferred to their respective arms. We had found the weights of the arms with the motors mounted in the correct position and used a general applied force of 20N. This simple static analysis allowed us to get started with our individual analysis. From these results the moment at the first arm was found to be 13.19748Nm, using this formula $M = F * d$, which is force (20N) times the distance from the base. By taking this number we can use it to find my max bending stress of the arm.

Max Bending Stress

Next step was calculating the second moment of area using the formula J=π/2 *(c_2^4-c_1^4) [4]. Which has been modified for using centroids of the inner and outer diameter instead of just using the outer and inner diameter. Where c_1= .0082804m for the inner diameter and c_2= .009525m for the outer diameter. When plugged into the equation we get J=5.544879x$10^{-9}m^4$. Now we have all we need to calculate max bending stress.

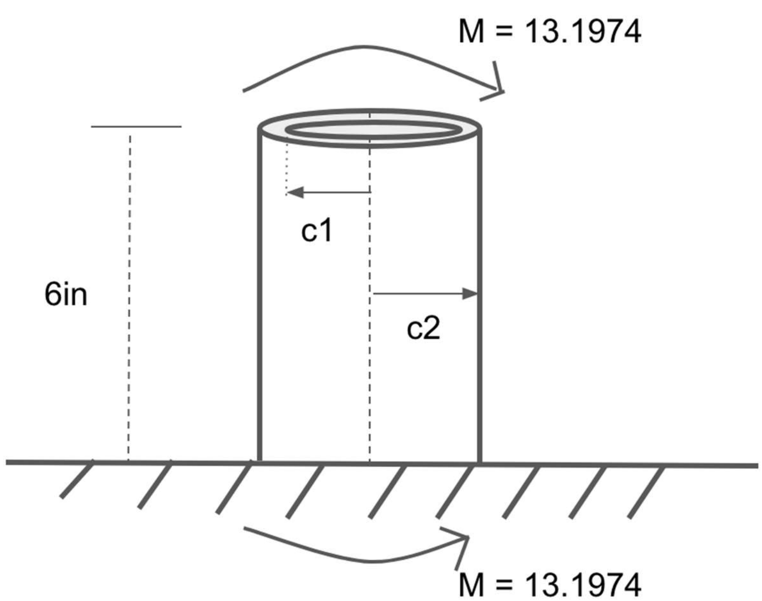


***Figure 8**: First link FBD*

$$\sigma = (F*c) / J$$

$$\sigma = (13.19748 Nm * .009525m) / 5.54487x10^{-9} m^4$$

$$\sigma = 22670684 \frac{N}{m^2} \approx 3288.105psi$$

The reason for choosing c to be equal to .009525m is because that is the farthest distance from the centroid of the pipe and will wear max bending stress. Again, with the yield stress of the pipe being 35000 psi we get a factor of safety of 10.6. This to reiterate is the worst-case scenario as we are not allowing the pipes to move, this would simulate if the design experienced a catastrophic failure of binding up during use.

Max Shear Stress

With the first link max shear was also calculated, again if the robot was to bind during use. The model being used is static to show the worst-case scenario for this stress analysis. Starting by finding the torque in the rod of the first link based on our initial force which was 20N. Then consider the position of this force based on the lengths of the links of the robot in a 45-degree angle for the second link, as shown in the diagram below. This is a top-down view of the robotic arm at a 45-degree angle with perpendicular force of 20N and highlighted in yellow is the pipe we will be finding torque for.

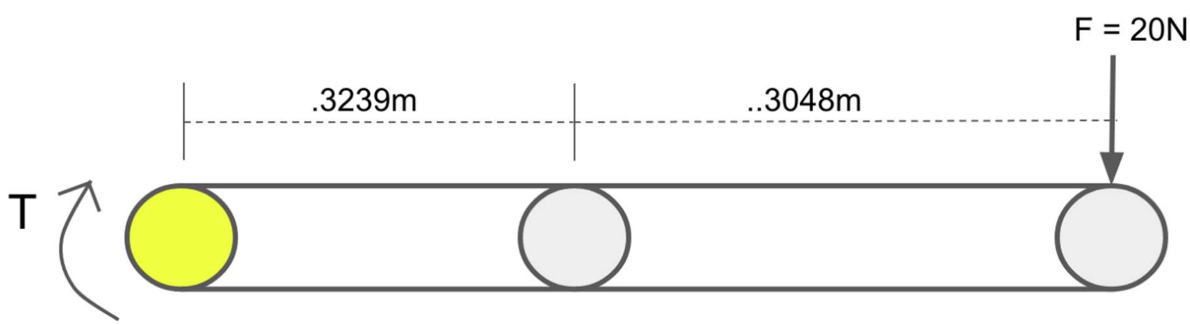


***Figure 9**: First Link Top View FBD*

The total distance for the base of the first link to be .6287 meter by using simple geometry of the triangle the second arm produced with the 20N applied force at the end. Using this formula T = F*r with F being 20N and r being .6287 as the distance from the base. The final number for the torque is 12.574Nm which after finding torque you can find the second moment of inertia. Using the formula from earlier, J = π/2 *(c_2^4-c_1^4) [4] where c_2 and c_1 are the centroid of the outer radius and inner radius respectively. This accounts for the hollow aluminum tube we have decided to use. When using this formula, we find that second moment of inertia is J=5.54487x$10^{-9} m^4$. Converting the length of the inner and outer radius from inches to meters we get c_1=.0082804m and c_2=.009525m. Now that we have enough information, we can use the formula to find max shear in the pipe. This formula τ=(T*c_2)/J where we will be using c_2 as that is the farthest distance from the centroid of the pipe and will be where the max shear in the pipe occurs.

$$\tau = ((12.574\text{Nm}) *(.009525\text{m})) / (5.54487 \text{ x } 10^{-9} m^4$$

$$\tau = 21599660 \frac{N}{m^2} = 3132.767 \text{ psi}$$

This number came out to well under our max yield stress for the aluminum pipe, which is 35000 psi. So, from this worst-case scenario if the bearing were to bind and not move at all, the applied stress would not cause the link to shear. This leaves us with a factor of safety of 11.2 which is good to know as now we understand that the shear the rod will experience in any direction is capable of handling it without a problem.

The Second Link

Looking into the second link, the aluminum rod as well as the pulley shaft are considered most important. The static analysis for these two parts assumes a worst-case scenario respectively. The aluminum arm will be able to spin freely after a load of around 40 N is applied perpendicular to the links when they are extended. This is because the motors, with pulley reductions, are able to have an increase of torque allowing the maximum to be around 30 Nm. After this moment is applied, the motor will slip/spin freely. For the pulley shaft, the worst case is when force is applied to the end of it far from where it is fixed into the system. Below are the breakdowns of these two important components. There are also a few assumptions for these calculations to consider.

Assumptions include:

- Static setting
- The aluminum rod has become completely rigid up through 20.0N
- Both robot arms are turned horizontal for maximum moment
- Arm length has increased to 18 inches
- Forces in shaft come from pulley analysis (400N)
- Arms are hollow
- All material properties are from MATWEB online and are accurate
- All failure will occur at weakest points

The Aluminum Rod

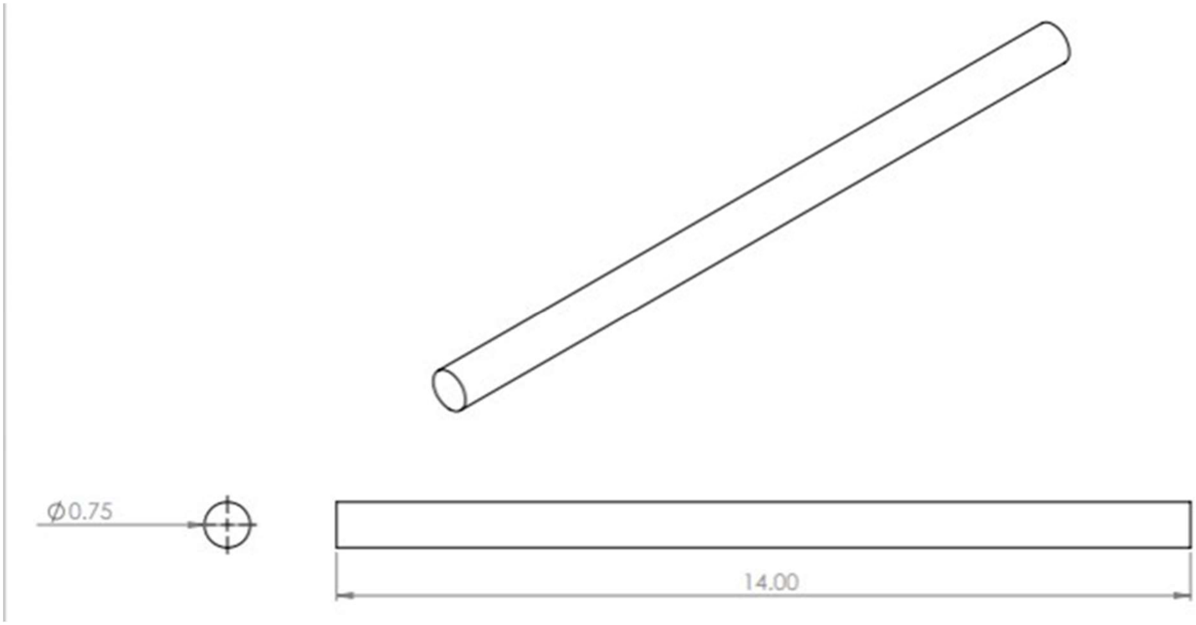


***Figure 10**: Aluminum rod [Inches] (NOTE: Assumed length will be 18 inches and the pipe is hollow)*

The rod itself presides with the first joint holding the bottom of it, allowing for at least 180-degree rotation. It can experience a force down the shaft if free movement stops for any reason. For this analysis the worst-case scenario is considered, where the rod length is actually 18 inches, the second rod is 12 inches, and both rods are hollow. The internal diameter is listed below as well. The equation and calculations for this are:

Shear stress

Diameters of pipe:

Outer DO = 0.01905 (m)   Inner di = 0.0165608 (m)

Area of pipe= pi*[((0.01905/2)m)^2 – ((0.0165608/2)m^2)]

Area = A = 6.962E-5 (m^2)

Stress = (F/A)

Stress = 20(N)/6.962E-4 (m^2) = 0.2873 (MPa)

Yield stress of Aluminum = 276 (MPa @ 24 degrees C)

Factor of safety [Shear] = (276)/(0.2873) = <u>960.67</u>

Bending stress

F = 20 (N)

L = 0.4572 + 0.3048 (m)

Moment = M = (F*L) = 15.24 (Nm)

Bending stress = (32*M*DO)/(pi*(DO^4-di^4))

= 0.052359 (GPa)

Factor of safety [Bending] = (276)/(0.052359) = 5271.3

Modulus of elasticity = E = 68.9 (GPa)

Moment of inertia = I = pi*(DO^4-di^4)/64 = 2.77E3

Deflection = (F*L^3)/(3*E*I)

Deflection = 1.543E-5 mm

Aluminum Rod Conclusions:

The yield stress is extremely low and will not be considered a problem to worry about for the rod. The aluminum rod will provide no problems for the robot. It should never have any problems moving since the robot will allow movement unless under motor control. The motors should fail if the force of 20 (N) is about doubled on the robot, which will allow all other parts to move freely. This rod is well beyond strong for the EG requirements.

The Joint Shaft

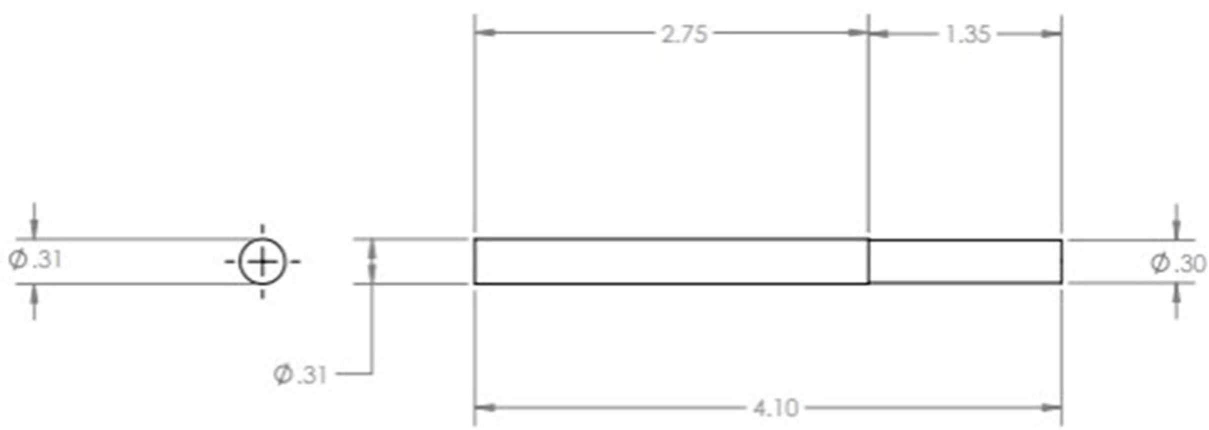


***Figure 11**: Steel Pulley Shaft [Dimensions are in inches]*

The shaft being considered holds the 100mm pulley, and has the highest amount of force put on it compared to any other part in the entire robot. This material is going to be made out of 316 steel and it has 2 different diameters, so the smallest diameter is considered for the analysis only. The forces on this shaft are derived from the pulley analysis and all calculations are listed below:

Shear stress

Force = F = 400 (N)

Area = A = pi*(0.00762/2 (m))^2 = 4.56E-5 (m^2)

Shear stress = (F/A) = 400 (N)/ 4.56E-5 (m^2)

= 8.77 (MPa)

Bending stress

Force = 400 (N)

L = 0.10414 (m)

Moment = (F*L) = 41.656 (Nm)

Area moment of inertia = I = pi*(D^4)/64

Bending stress = M*r/I

= 0.959 (GPa)

Mass = m = 8000(kg/m^3)*A*L = 0.038 (kg)

Moment of inertia = I = m*[((r^2)/4)+((L^2)/12)] = 7.346

Modulus of elasticity = E = 193 (GPa)

Deflection = (F*L^3)/(3*E*I)

= 1.06E-10 (mm)

Joint Shaft Conclusions:

The joint has high force applied to the cross-sectional area, but it is able to easily handle such loads. Looking at the shear stress, the yield is well above the found stress on the shaft, it is going to be fine. The bending stress is also about 0.5% of the modulus of elasticity, so that will also be completely fine. The total deflection is extremely small, so even if the calculations are off by 10^9, the shaft will still not deflect more than 1/10 of a mm. It will not even be detectable so all the calculations prove that the joint should be completely fine since the weakest points in the worst scenarios prove to be well below the allowed maximum.

The Third Link

The analytical process of all links shares a common analysis of static forces acting on the links as well as the torque forces resulting from the robot movement based on a 20 N point load acting on the end effector/handle of the robot. In this section, the essential analyses done on the third link are presented to prove the complete functionality of the link is within the client and engineering requirements set by the team. The focus of this section is to show that the stresses endured on the third link and on the single joint connection on the link are indeed stiff, safe, and have met the requirements of the design. A basis of static force analysis is established by the team to be able to move further with the design analyses of each link and main subsystems of the robot. Therefore, a static, stress, and deflection analyses on the third link and specifically the third joint connection is expressed in the following section including equations and calculations of the process.

Static Analysis

The team used equations 1,2, and 3 in order to figure out the sum of the forces on the robot due to the 20 N axial force that is applied to the handle.

$$Fy = 0 \tag{4}$$

$$Fx = 0 \tag{5}$$

$$M = 0 \tag{6}$$

The results from calculating the forces on the robot when it's in a weaker position of *45°* and a horizontal position for the third link as a distributed load W3 which represents the weight of the third link and it's equal to *0.0573 kg*. The third link is considered the second heaviest of all three links on the robot. The only torque that is taking place on the third link is at the third joint due to the joint connection between link two and three. The torque is calculated as part of the static analysis based on the 20 N force applied to the robot. The resulting torque/moment from using the above equations is equal to *6.1047 Nm* which is the lowest torque in all of the three joints of the robot. The result is reasonable since the third link has only one connection that is exposed to a torque force as opposed to the other two links that are exposed to two different torque loads on each link.

Stress Analysis

Based on the values that resulted from the static analysis, the forces will be used to implement into the stress equation 4 [1] which focuses on the normal stress over the entire cross section of the third link to determine how much force will be applied to the link due to the 20 N horizontal force.

$$\text{Normal Stress: } = F/A \tag{7}$$

Using 20 N as the force (F) in equation 4 and 6.9597E-5 m^2 for the area (A) in the equation results in the normal stress on the third link to be *0.2874 MPa*. Knowing that the stress is very low on the third link justifies the safety and stiffness of the link to satisfy the client requirement. Another type of stress that needs to be considered on the third link is the shearing stress on the third joint caused from the 20 N force as well. Equation 5 [1] provides the maximum shear stress on a given cross section due to a given load. In this formula, the shear force (V) represents the 20 N force on the joint while the cross section represents the area (A) of the joint.

$$\text{Shear Stress: } 2V/A \quad (8)$$

After using 20 N for (V) in equation 5 and 100π mm^2 for the area of the joint (A) the result is equal to the maximum shear stress on the joint which is *0.1273 MPa*. The shear stress value is indeed very low giving the joint a high factor of safety and a reliable result for the team to continue using the same joint material. As shown in Figure 1, the yield strength of the joint undergoing 20 N of force is about *27.5 MPa* which gives a factor of safety of over *200* based on the material used for the joint, and as shown in **Table 2** the factor of safety based on the Finite Element Analysis on the joint is 4070 which validates the calculations made above.

Strain & Deflection Analysis

In order to analyze the deflection of the third link we'd have to use the available tables for the different beam deflection types and since the third link is considered fixed at the handle and exposed to a moment of torque at the other end by the joint connection, the cantilever beam is therefore used as the type of beam being analyzed and equation 6 [2] represents the deflection of the beam.

$$\delta \max = \frac{ML^2}{2EI} \quad (9)$$

The variables M, L, E, and I represent the moment on the other end of the fixed point, the length of the link, the modulus of elasticity of the link, and the moment of inertia of the link respectively.

The values of the four variables are 6.1047 Nm, 0.3048 m, 70E9 Pa, and using $I = \pi D^4/64$ with a diameter of 0.652 m to be 8.87E-3 m^4 respectively. The result of the maximum deflection of the link comes out to be *4.5667E-10 m* which is a very low deflection in the link, therefore, justifying another important design necessity based on the client needs.

Another important calculation for the third link's design analysis is the total engineering strain on the link which is presented in equation 7 [3] where the total strain is equal to the maximum deflection calculated above divided by the original length of the link.

$$\varepsilon = \frac{\delta}{L} \tag{10}$$

To calculate the strain in equation 7, the maximum deflection value is 4.5667E-10 m and the total length (L) of the link is 0.3048 m resulting in the value of *1.49827E-9* of strain on the link. Based on this calculation, the client can be assured that the third Aluminum link that is used on the robot which is subjected to a maximum force of 20 N has a very low strain, and therefore, meets the requirement of stiffness and is okay to use for implementation moving forward with the design process.

Table 2: Sub-system parts Load Cases and resulting Factors of Safety

Sub-system	Part	Load Case Scenario	Material	Minimum FoS
Motor Mounts				**1.1**
	L-Bracket 1	Maximum of 20 N force on the bracket from the pinion and motor shaft.	Alloy Steel	6821
	L-Bracket 2 (small)	Load from pinion and 60 mm pulley.	Alloy Steel	1283
	L-Bracket Motor 2	Load on bracket from motor and clamps attached to the link.	Alloy Steel	2020
	L-Bracket 1st Link (small)	Load on bracket from 10 mm pinion and the second 60 mm pulley.	Alloy Steel	1924
Pulleys				
	Pinion Pulley 1	Load on pinion from Steel Cables.	Plastic ABS (3D Printed)	<1
	10 mm Pinion	Load from Steel Cables between driven pulley 1.	Plastic ABS	< 1
	Pinion Pulley 2 (Link 1)	Load coming from Steel Cables between 60 mm Pulley on the 1st Link.	Plastic ABS	< 1
	Pinion Pulley 3 (Link 2)	Load from Steel Cables coming from the top 150 mm driven pulley	Plastic ABS	< 1
	60 mm Pulley	The load coming from the motor on the shaft coupler is the concerning point.	Plastic ABS	< 1
	Bottom Driven Pulley	The load coming from the coupler above and bearing below on the pulley causing deformation.	Plastic ABS	< 1

	60 mm Pulley (2)	One way load coming from the shaft coupler on the pulley is the main concern as well as the deformation of the pulley from Steel Cables.	Plastic ABS	< 1
	100 mm Pulley	Shear force on shaft coupler connected to the 2nd joint connection as well as deformation caused from Steel Cables on pulley.	Plastic ABS	< 1
	150 mm Pulley	Shear force caused by Steel Cables pulling and shaft connection from joint 3 to the pulley.	Plastic ABS	< 1
Links & Joints				
	Link 1	The shear force on the top of the link caused by the 2nd joint connection and normal force caused by the 20 N applied from the end-effector.	Aluminum 1060	174
	Joint 1	Force applied on the joint by the rod as well as the bolts connections.	Alloy Steel	4832
	Link 2	Force on the rod from top caused by the 20 N coming from the 3rd joint connection as well as a normal force coming from the 1st joint connection.	Aluminum 1060	510
	Joint 2	Same as joint 1, forces transmitting through the joint can cause deflection of joint as well as shear from bolts on the rod.	Alloy Steel	4070
	Link 3	20 N force coming from the end effector could cause shear at the top as well as the bottom of the rod from joint connection.	Aluminum 1060	179

Flow Charts and other Diagrams

Below is our functional model we use to describe the steps our robot takes to achieve its required task. We have four inputs which are human, controller, electricity and the hand. Also included are the outputs for our robot such as thermal energy, protentional energy, kinetic energy, haptic feedback and the hand again. This chart shows how each input relates to an output, some of which connect to multiple outputs.

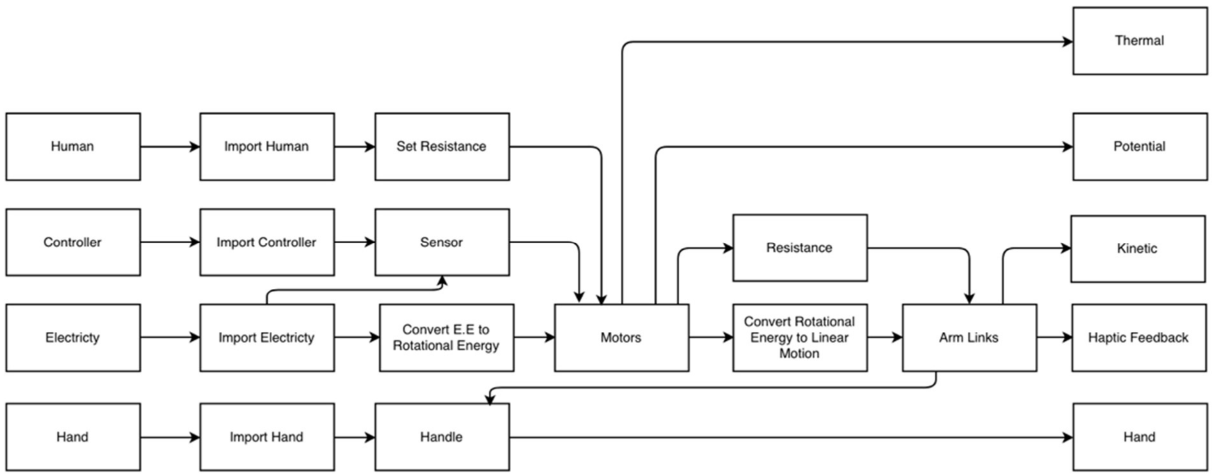


Figure 12: Functional Model

Moving Forward

Moving forward, the team has plans to make a few design changes which would require us to make a couple more calculations. This includes a torque calculation based on the new joint we plan on using and a different shape for the links which would change the weight in the force analysis. The pulley systems are mainly done other than making sure we are analyzing the cable of on the reel once the pulley system is designed to our client's needs. In addition, it is apparent that the resulted factor of safety for all pulleys are not acceptable values, therefore, the team is taking a step into developing a stronger material for the pulley systems to ensure stiffness and overall safety of the design functionality.

- Speak on motor brackets (shouldn’t be a problem)
- Speak on pulleys strength (very low factor of safety)

References

[1] D. Wallace, “Basic Stress Equations.” Available: http://www.faculty.fairfield.edu/wdornfeld/ME311/BasicStressEqns-DBWallace.pdf

[2] “BEAM DEFLECTION FORMULAE BEAM TYPE SLOPE AT FREE END DEFLECTION AT ANY SECTION IN TERMS OF x MAXIMUM DEFLECTION.” Available: https://home.engineering.iastate.edu/~shermanp/STAT447/STAT%20Articles/Beam_Deflection_Formulae.pdf

[3] “Engineering Stress/Strain vs True Stress/Strain – Yasin ÇAPAR.” https://yasincapar.com/engineering-stress-strain-vs-true-stress-strain/

[4] “Area moment of inertia - typical cross sections I,” Engineering ToolBox, https://www.engineeringtoolbox.com/area-moment-inertia-d_1328.html (accessed Aug. 4, 2023).

[5] “McMaster-Carr,” *www.mcmaster.com*.https://www.mcmaster.com/2655N19/ (accessed Aug. 05, 2023)

Appendices

Appendix A – QFD

Project:	Haptic Robot
Date:	June 20th, 2023

System QFD

#	Technical Requirement	Decrease Weight	Tolerance	Material Strength	Force	Reduce Friction	Speed
1	Decrease Weight						
2	Tolerance	1					
3	Material Strength	-3	1				
4	Force		1	9			
5	Reduce Friction	9	1		1		
6	Speed	9	1	-3	-9	9	
7	Electrical Power	1	1		-9		-9
	Range of Motion						

Legend		
A	Wooden Haptics	https:// woode
B	HIRO II	https:// www.in
C	Haptx	https:// haptx

#	Customer Needs	Customer Weights	Decrease Weight	Tolerance	Material Strength	Force	Reduce Friction	Speed	Electrical Power	1 *Poor*	2	3 *Acceptable*	4	5 *Excellent*
1	Lightweight	5	9	3	9		1	9	1		C	B		A
2	Affordable	3			1				1	BC		A		
3	User Friendly	3	3				3	9						ABC
4	Stiff	5		3	9	3	1	3			A	B		C
5	Accurate	4	3	9			3	3	9			AB		C
6	Flawless Motion	5	3			1	9	9	3		B	A		C
7	No Backlash	5	9	9	3	3	3	9	1				AB	C
8	No Friction	4	9			9	9	9			B		A	C
9	Produce Force	5	3		9	9	1	9	9	B		A		C
	Technical Requirement Units		lbs	%	Gpa	Newtons	Newtons	m/s	Watts					
	Technical Requirement Targets		50	1	2	20	1	1	100					
	Absolute Technical Importance		162	93	120	110	129	261	107					
	Relative Technical Importance		2	7	4	5	3	1	6					

Technical Requirements: Decrease Weight – Electrical Power; Customer Opinion Survey: 1–5.

Appendix B – Given Diameters and Driver Speeds

```
%Given Values
D1 = 150 %mm Diameter of Pulley 1
D2 = 10 %mm Diameter of Pulley 2
D3 = 100 %mm Diameter of Pulley 3
D4 = 60 %mm Diameter of Pulley 4
D5 = 10 %mm Diameter of Pulley 5
D6 = 10 %mm Diameter of Pulley 6
Spd2 = 2000 %rpm Pulley 2 Driver Speed
Spd6 = 2000 %rpm Pulley 6 Driver Speed
```

Appendix C – Base and Link 1 Pulley MATLAB Analysis

```
%Base and Link 1 Pulley
RatioS1 = D4/D6 %Ratio of Pulley Set 1
RatioS2 = D3/D5 %Ratio of Pulley Set 2
TotRatio = RatioS1*RatioS2 %Total Pulley Ratio

Spd4 = Spd6/RatioS1 %rpm Speed of Pulley 4
Spd5 = Spd4 %rpm Speed of Pulley 5
Spd3 = Spd4/RatioS2 %rpm Speed of Pulley 3
```

```
RatioS1 = 6
RatioS2 = 10
TotRatio = 60

Spd4 = 333.3333
Spd5 = 333.3333
Spd3 = 33.3333
```

Appendix D – Link 2 Pulley MATLAB Analysis

```
%Link 2 Pulley
RatioL2 = D1/D2 %Ratio of Pulley Set
Spd1 = Spd2/RatioL2 %rpm sdSpeed of Pulley 1
```

```
RatioL2 = 15
Spd1 = 133.3333
```

Appendix E – Link 1 (Pulley 1)

```
T = 20;
M = 13.2;
r = .05;
C = T/r;
Q = M/r;
v = C + Q;
T1 = v/2;
T2 = C-T1;
fprintf('The Initial Tension is')
disp(C)
fprintf('The moment divided by radius is')
disp(Q)
fprintf('The added value is')
disp(v)

disp('Final Answers')

fprintf('T1 is Equal to')
disp(T1)
fprintf('T2 is Equal to')
disp(T2)
```

ommand Window

lew to MATLAB? See resources for Getting Started.

```
>> PulleyC
The Initial Tension is   400

The moment divided by radius is  264.0000

The added value is   664

Final Answers
T1 is Equal to   332

T2 is Equal to    68
```

Appendix F – Link 1 (Pulley 2)

```
T = 20/10;
M = 13.2/10;
r = .03;
C = T/r;
Q = M/r;
v = C + Q;
T1 = v/2;
T2 = C-T1;
fprintf('The Initial Tension is')
disp(C)
fprintf('The moment divided by radius is')
disp(Q)
fprintf('The added value is')
disp(v)

disp('Final Answers')

fprintf('T1 is Equal to')
disp(T1)
fprintf('T2 is Equal to')
disp(T2)
```

ommand Window

ew to MATLAB? See resources for Getting Started.

```
>> PulleyC
The Initial Tension is   66.6667

The moment divided by radius is   44.0000

The added value is  110.6667

Final Answers
T1 is Equal to   55.3333

T2 is Equal to   11.3333
```

Appendix G – Link 2 (Pulley 1)

```
T = 20;
M = 13.0019;
r = .05;
C = T/r;
Q = M/r;
v = C + Q;
T1 = v/2;
T2 = C-T1;
fprintf('The Initial Tension is')
disp(C)
fprintf('The moment divided by radius is')
disp(Q)
fprintf('The added value is')
disp(v)

disp('Final Answers')

fprintf('T1 is Equal to')
disp(T1)
fprintf('T2 is Equal to')
disp(T2)
```

ommand Window

ew to MATLAB? See resources for Getting Started.

```
>> PulleyC
The Initial Tension is   400

The moment divided by radius is  260.0380

The added value is  660.0380

Final Answers
T1 is Equal to  330.0190

T2 is Equal to   69.9810
```

Appendix H – Link 2 (Pulley 1)

```
T = 20;
M = 6.1047;
r = .075;
C = T/r;
Q = M/r;
v = C + Q;
T1 = v/2;
T2 = C-T1;
fprintf('The Initial Tension is')
disp(C)
fprintf('The moment divided by radius is')
disp(Q)
fprintf('The added value is')
disp(v)

disp('Final Answers')

fprintf('T1 is Equal to')
disp(T1)
fprintf('T2 is Equal to')
disp(T2)
```

Command Window

New to MATLAB? See resources for Getting Started.

```
>> PulleyC
The Initial Tension is  266.6667

The moment divided by radius is   81.3960

The added value is  348.0627

Final Answers
T1 is Equal to  174.0313

T2 is Equal to   92.6353
```

Appendix I – Link 3 Pulley

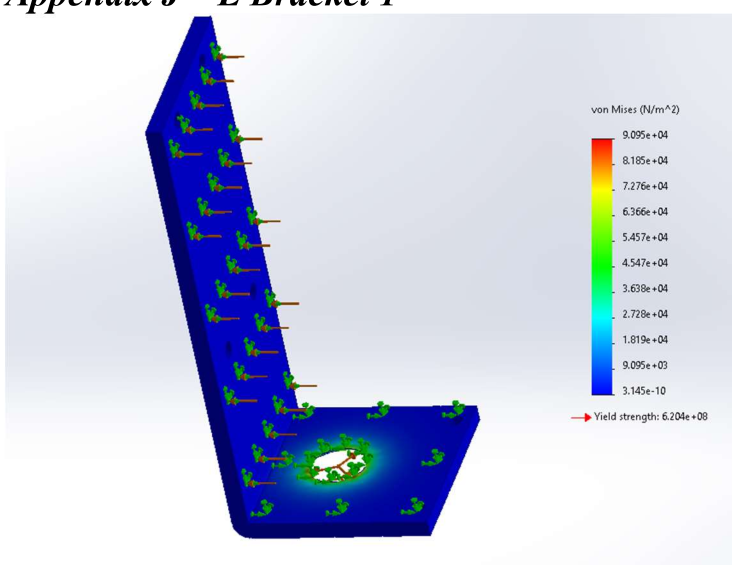
```
T = 20/10;
M = 13.0019/10;
r = .03;
C = T/r;
Q = M/r;
v = C + Q;
T1 = v/2;
T2 = C-T1;
fprintf('The Initial Tension is')
disp(C)
fprintf('The moment divided by radius is')
disp(Q)
fprintf('The added value is')
disp(v)

disp('Final Answers')

fprintf('T1 is Equal to')
disp(T1)
fprintf('T2 is Equal to')
disp(T2)
```

ommand Window

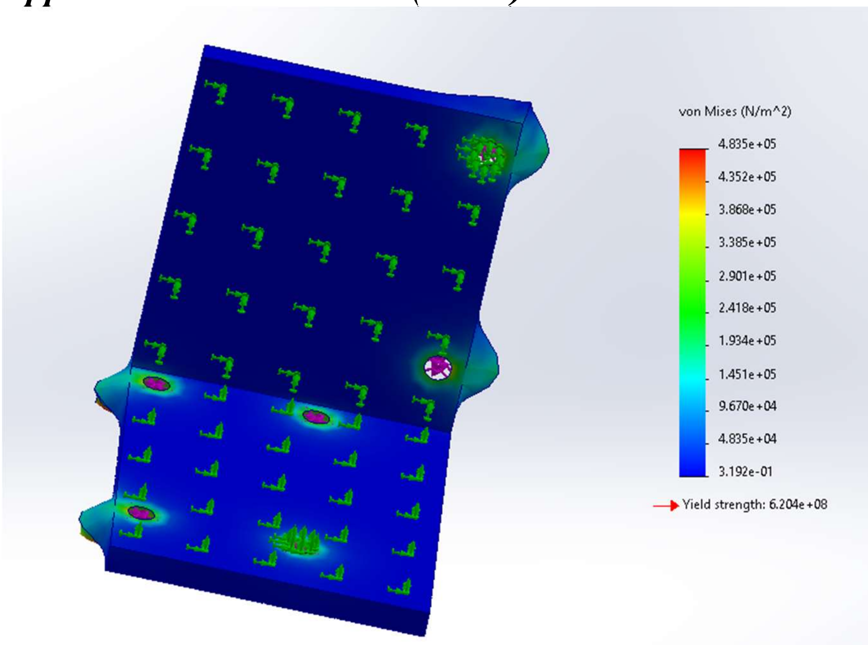
lew to MATLAB? See resources for Getting Started.

```
>> PulleyC
The Initial Tension is   66.6667

The moment divided by radius is   43.3397

The added value is  110.0063

Final Answers
T1 is Equal to   55.0032

T2 is Equal to   11.6635
```

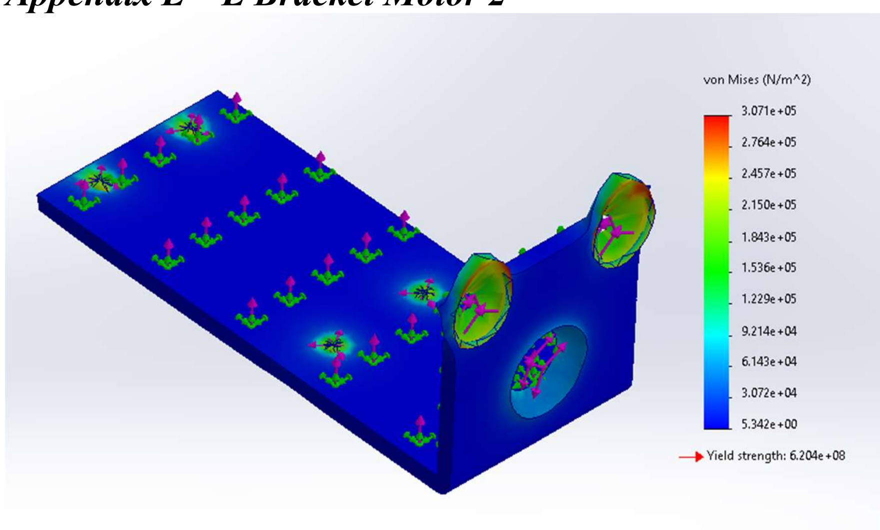
Appendix J – L Bracket 1



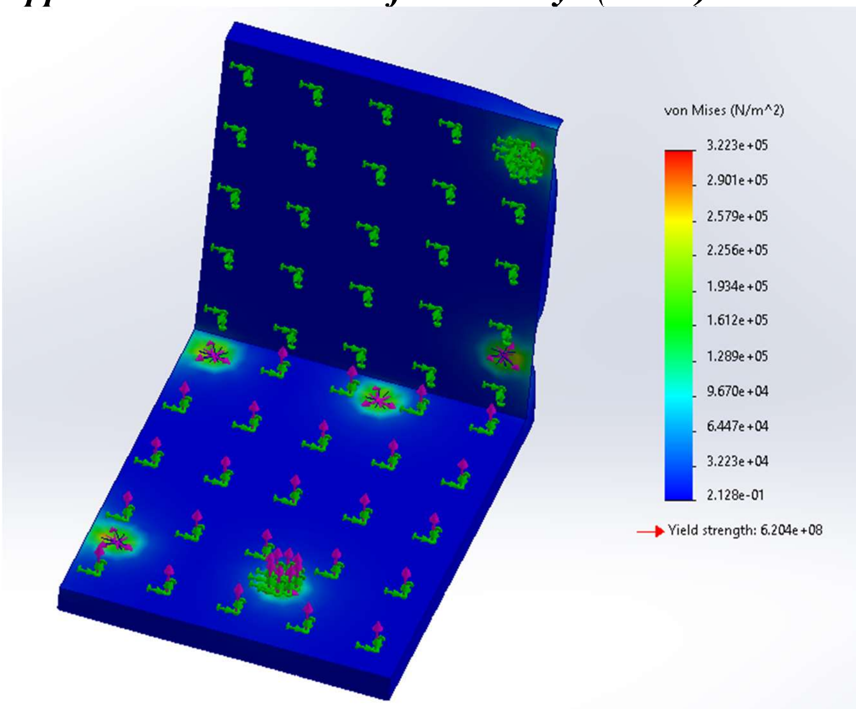
Appendix K – L Bracket 2 (Small)



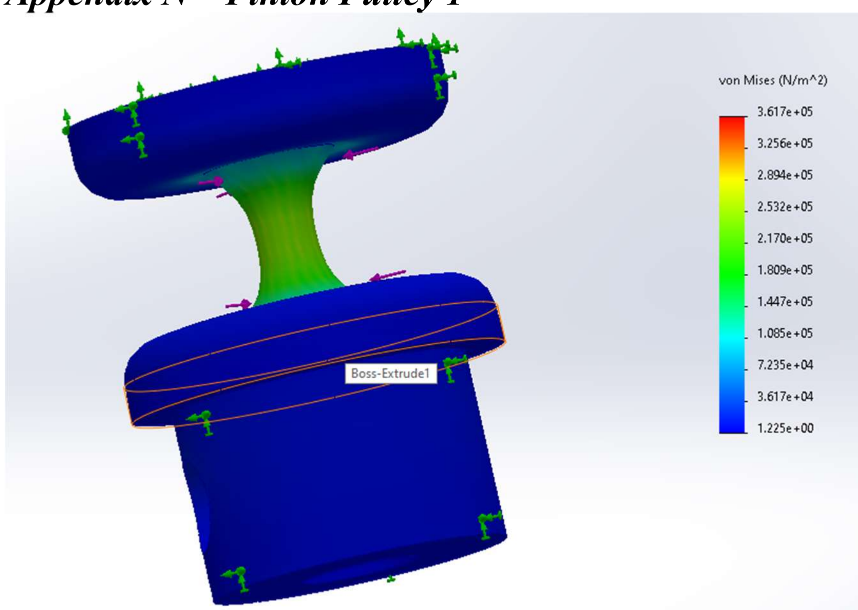
Appendix L – L Bracket Motor 2



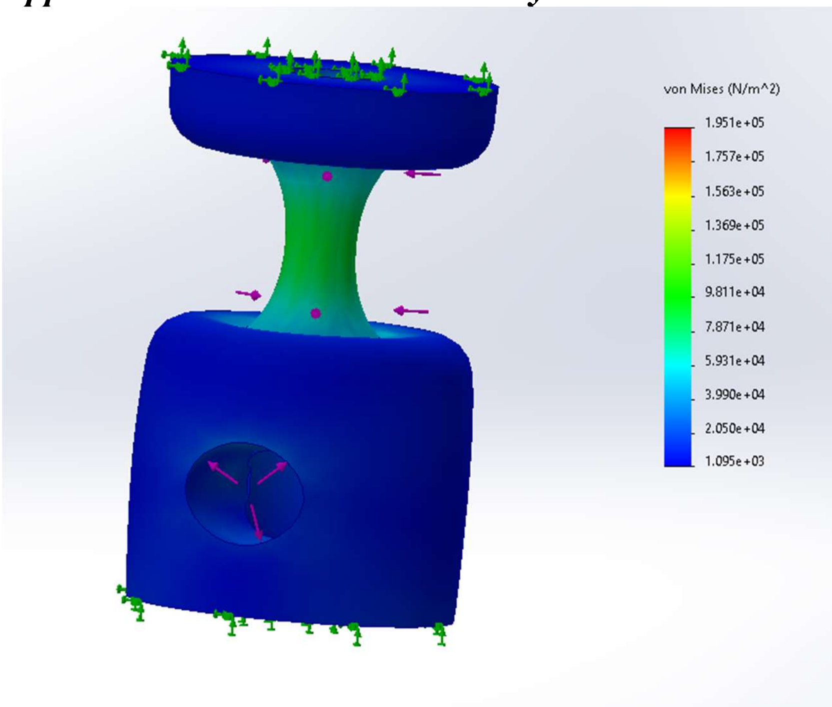
Appendix M – L Bracket for 2 Pulleys (Small)



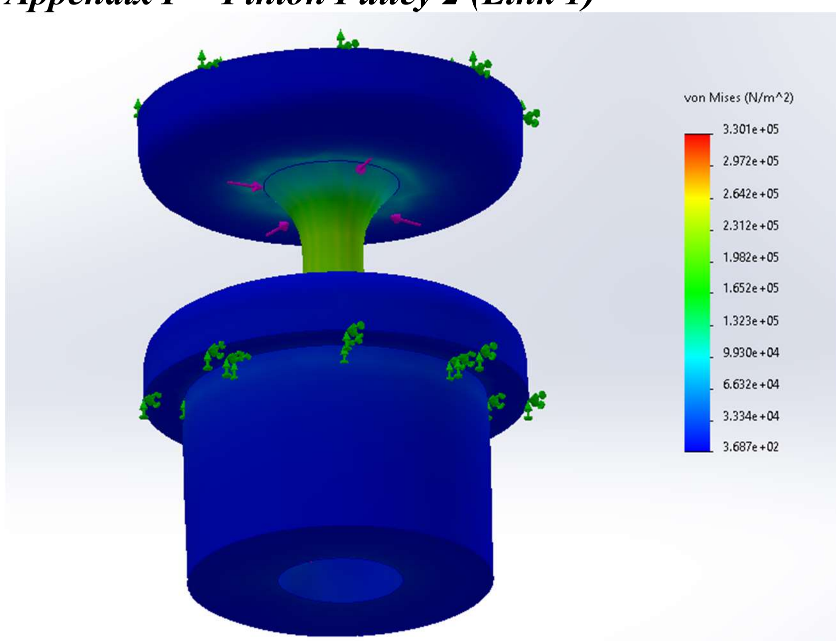
Appendix N – Pinion Pulley 1



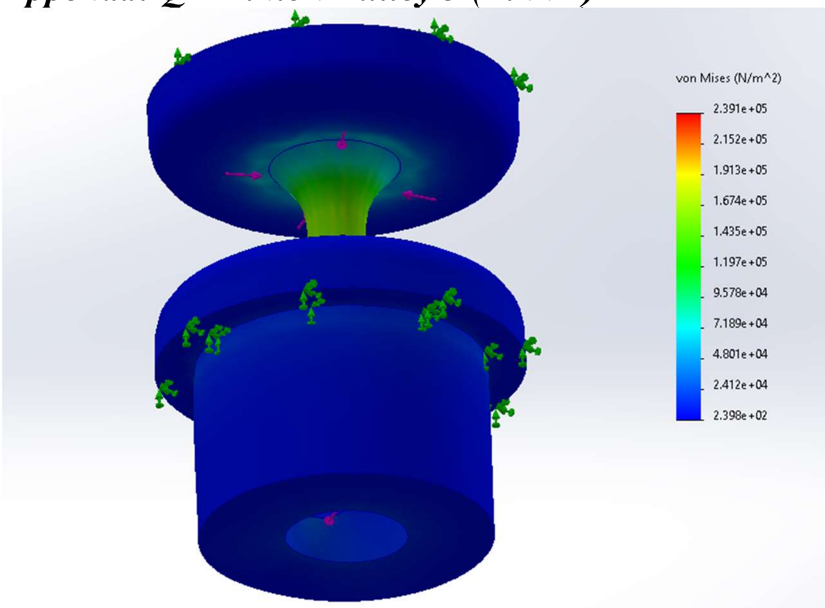
Appendix O – 10 mm Pinion Pulley



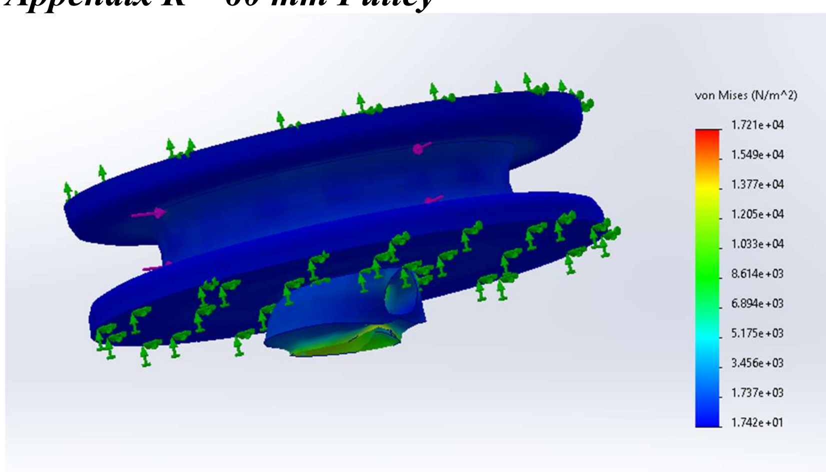
Appendix P – Pinion Pulley 2 (Link 1)



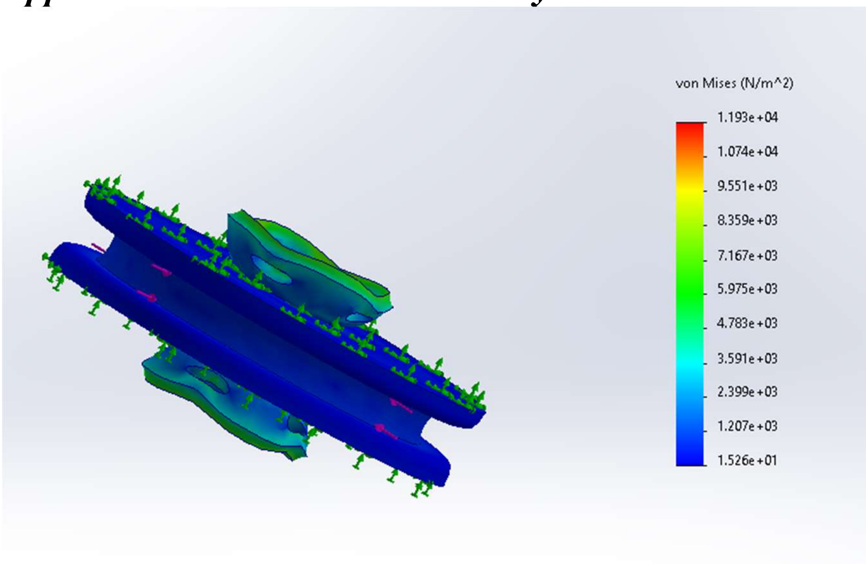
Appendix Q – Pinion Pulley 3 (Link 2)



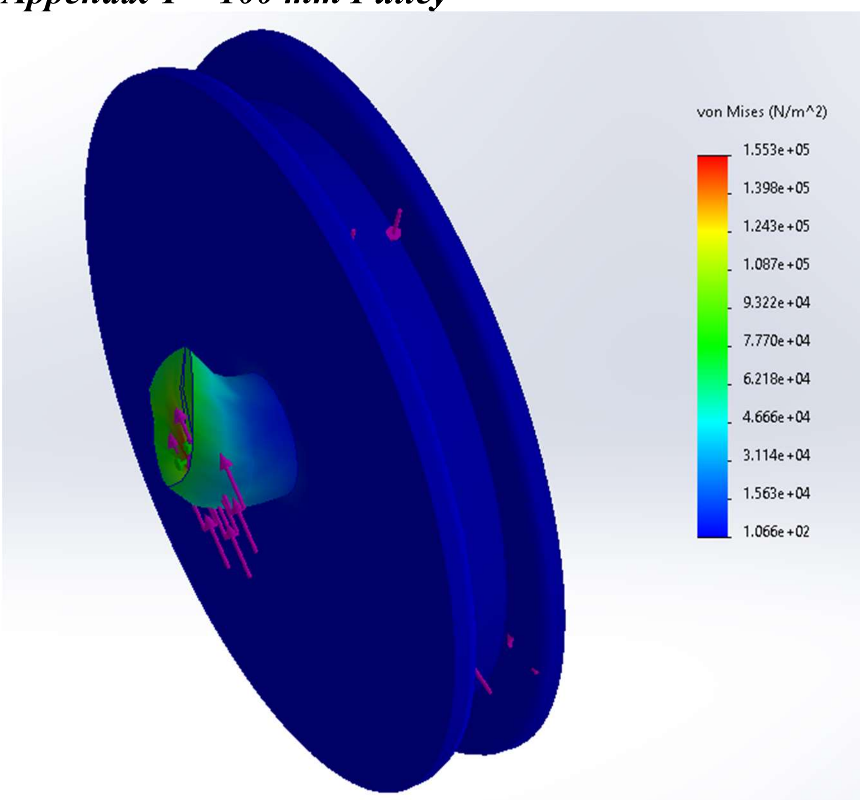
Appendix R – 60 mm Pulley



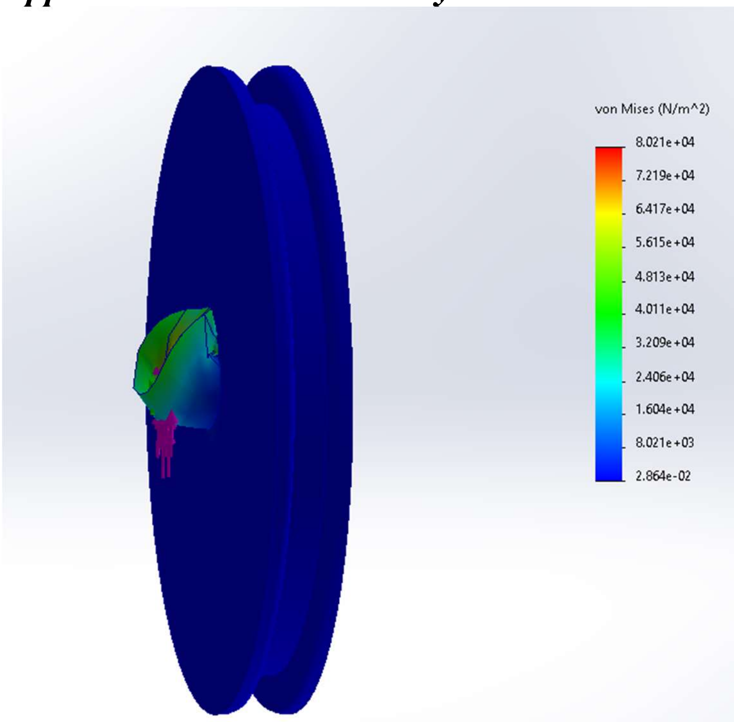
Appendix S – Bottom Driven Pulley



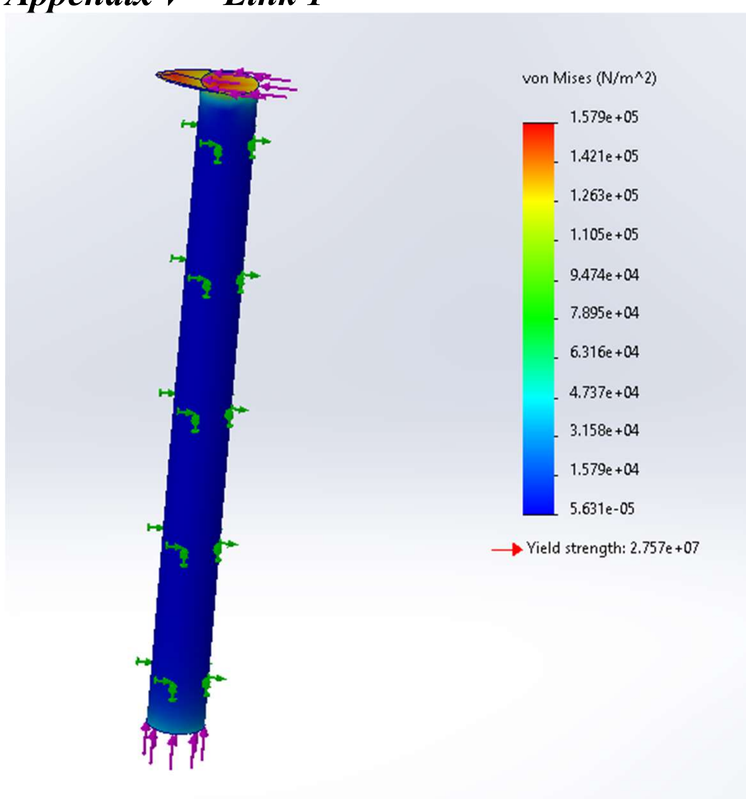
Appendix T – 100 mm Pulley



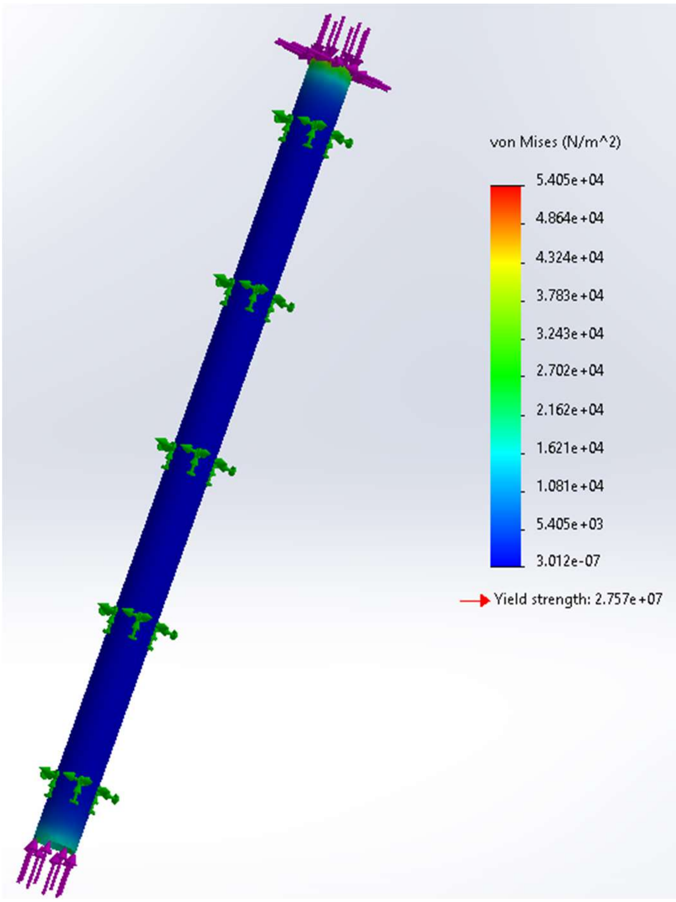
Appendix U – 150 mm Pulley



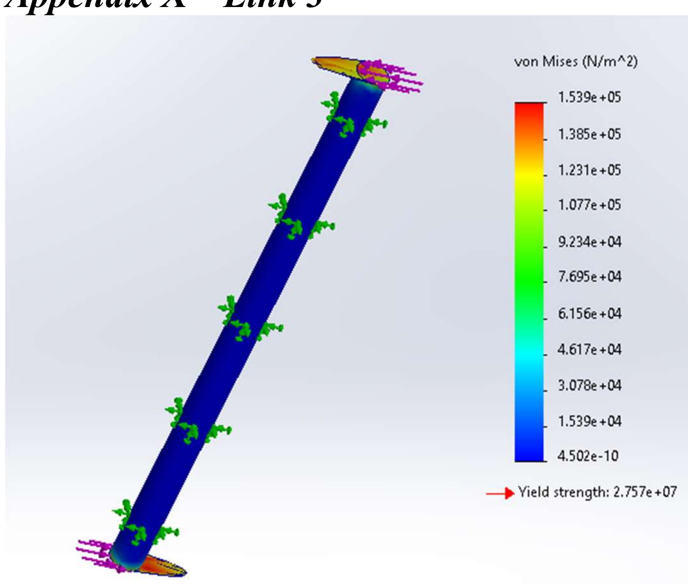
Appendix V – Link 1



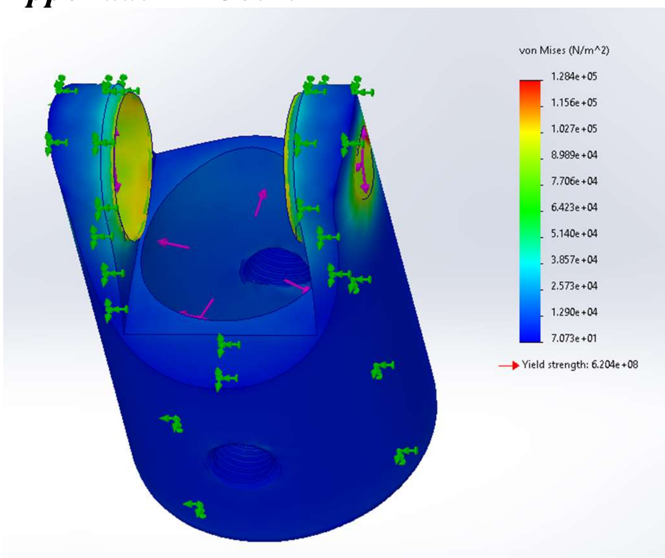
Appendix W – Link 2



Appendix X – Link 3



Appendix Y – Joint 1



Appendix Z – Joint 2