

**HPVCP Summer**

**Individual Analytical Analysis**

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**Section: ME 486C-001**

## **Introduction**

Human-powered vehicles are vehicles driven by human power. The specific specifications and shapes of human-powered vehicles are not the same. The most widely used basic structure in modern society is the bicycle. The bicycle is driven forward by manpower. The purpose of our group project is to design a unique human-powered vehicle. The team has been working on developing a regeneration propulsion system for the existing HPV. After two semesters of research, the team has designed a new propulsion system that relies on flywheels to store energy. The current design is a kinetic energy recovery system. We will retrofit the existing HPV selected by the customer.

For the propulsion system designed by the team, the transmission part is a very important part. The movement mode and mechanical efficiency of the whole vehicle are related to the transmission part. Our team divides the transmission part into specific details. I am responsible for the roller chain and sprocket. In my personal analysis during the summer vacation, I learned about the models and specifications of roller chains and sprockets and explored the sprocket transmission ratio. This semester's report research focuses on the characteristics and calculations of roller chains and sprockets, as well as the finite element analysis of the sprocket and the shaft used to support the connection.

## **Chain and Sprockets**

Let me briefly sort out the main points of the last individual analysis. Since I just came into contact with roller chains and sprockets during summer vacation and did not learn ME465-Machine Design II, I didn't have enough comprehensive and detailed understanding when studying sprockets and roller chains.

### Previous Summary

A complete roller chain is composed of rollers, bushes, pins, inner plates, and outer plates. The inner plate and the bushing, the outer plate, and the pin are respectively fixedly connected by small shafts. There are clearances between the roller and the bushing and between the bushing and the pin to add lubricating oil and prevent excessive friction. The wear of the chain mainly occurs on the contact surface of the pin and the bushing, so the presence of lubricating oil greatly reduces the friction of the chain. Making the inner and outer plates into a ring shape can make each surface of the chain have similar tensile strength while reducing the weight of the chain and the inertial force generated during displacement and rotation.

### Differences between analyses

The chain adopts the uniform size specified by the American Standard, and the label is ANSI. The sprocket data from ANSI#25 to #240 have been standardized and can be queried. [3] The most commonly used chain set for bicycle sprockets is ANSI#40. But we will use ANSI#35 chain and sprockets in our design. This is not only the same as the chain model of the car we need to modify, but also has a smaller weight and pitch diameter.

In the final design, the number of sprocket teeth is selected according to the transmission ratio. For example, a high transmission ratio sprocket will waste more energy but will increase the speed, suitable for male or professional riders.

ANSI Chain Number	Pitch, in (mm)	Width, in (mm)	Minimum Tensile Strength, lbf (N)	Average Weight, lbf/ft (N/m)
25	0.250 (6.35)	0.125 (3.18)	780 (3 470)	0.09 (1.31)
35	0.375 (9.52)	0.188 (4.76)	1 760 (7 830)	0.21 (3.06)
41	0.500 (12.70)	0.25 (6.35)	1 500 (6 670)	0.25 (3.65)
40	0.500 (12.70)	0.312 (7.94)	3 130 (13 920)	0.42 (6.13)

Figure 1: Dimensions of ANSI Roller Chains—Single-Stranded

Figure 2 below is from ME465 Machine Design II. [3] The basic parameters of roller chain and sprocket meshing are pitch  $p$ , roller outer diameter  $d_1$ , and inner link width  $b_1$ . [1] The pitch diameter is the main parameter of the roller chain. When the pitch diameter increases, the size of each part of the chain will increase accordingly.

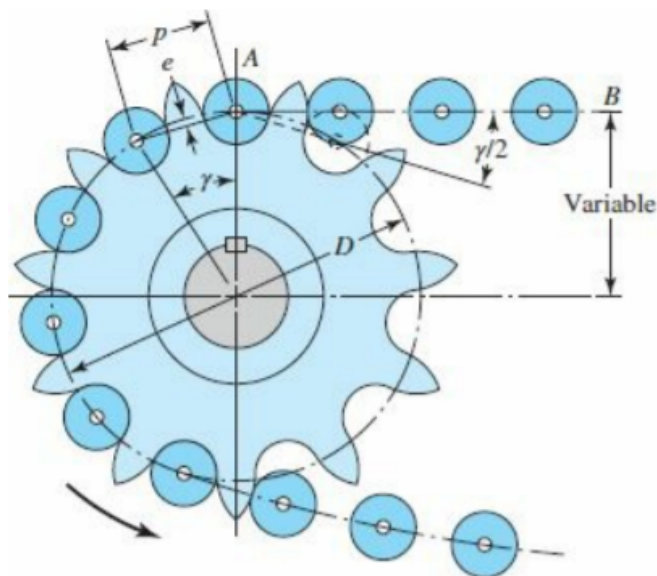


Figure 1: Engagement of a chain and sprocket. [3]

## Mathematical/Calculative analysis

### Velocity of Chain

The following formula 1.1 shows the relationship used when the chain rotates counterclockwise, where the sprocket drives the chain.  $p$  represents the chain pitch,  $\gamma$  represents the pitch angle, and  $D$  represents the diameter of the sprocket pitch circle:

$$D = \frac{p}{\sin(\gamma/2)}$$

(Equation 1.1)

Since  $\gamma=360^\circ/N$ , where  $N$  is the number of sprocket teeth, formula 1.1 can be written as the following formula 1.2:

$$D = \frac{p}{\sin(180^\circ/N)}$$

(Equation 1.2)

The angle  $\gamma/2$  at which the connecting rod rotates when it contacts the sprocket is called the joint angle, and the size of this angle is a function of the number of teeth of the gear. When the chain links rotate through this angle, friction between the rollers (bearings) and the sprocket teeth will be generated, which will promote the rotation of the roller chain and cause the wear of the chain links. Reasonable use of lubricating oil to reduce the friction of the rollers inside the chain link can greatly increase the service life of the chain. The speed of rotation of the chain,  $V$ , is the number of feet disengaged from the sprocket per minute. Therefore, the chain speed in feet per minute is:

$$V = \frac{Npn}{12}$$

(Equation 2)

where  $N$  = number of sprocket, teeth

$p$  = chain pitch, in

$n$  = sprocket speed, rev/min

In actual use, even if the driving sprocket rotates at a constant and unique angular velocity, its instantaneous speed and instantaneous transmission ratio will change at any time.

According to our design and the speed of the flywheel, we can calculate the angular velocity required for the rotation of our sprocket. This will help us verify the safety of the sprocket and chain during our actual use. The calculations are shown in Table 1 below.

Table 1: Calculated velocity of chain and Diameter of pitch

ANSI#	35	Number of teeth	24	Material	Carbon Steel
Pitch, in	0.375	Bore Pitch, in	0.5		
Width, in	0.188	Angular Velocity, rev/min	974.028		
Min Tensile Strength, lbf	1760				
Average Weight, lbf/in	0.21				
Diameter of pitch circle, in	0.35174999				
Velocity of chain, rev/min	730.521				

### Length and Number of Chain

The chain length is determined by the number of chain links and the distance between the chain rows. The trajectory of the chain drive depends on the position of the individual sprocket and its direction of movement. To calculate the chain length, the sprocket pitch circle diameter will be used, which is the pitch circle diameter calculated in the table above. The pitch circle diameter of each roller chain drive sprocket is calculated by the following formula. We can adjust the position of the sprocket accordingly to achieve the required chain length, or connect 5-15 more chain links to ensure that there is enough space to connect to the transmission system. More connecting links can also reduce the friction between the sprocket and the chain to a certain extent.

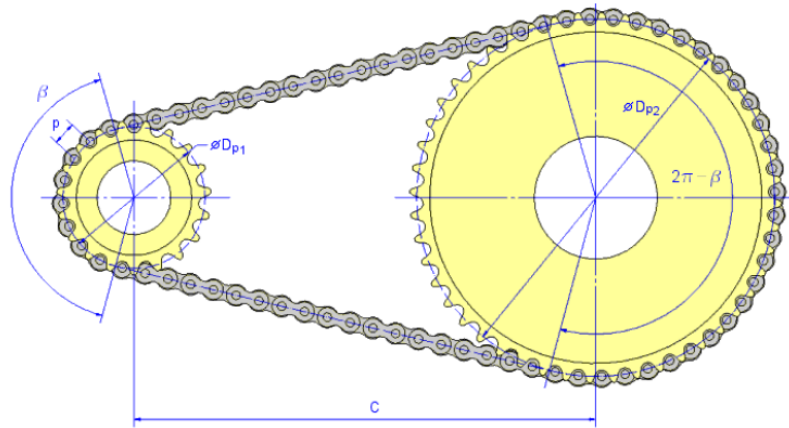


Figure 3: Chain drive

A number of chain links required for center distance X0:

$$X_0 = 2 \cdot \frac{C_0}{p} + \frac{z_1 + z_2}{2} + \frac{p \cdot \left( \frac{|z_2 - z_1|}{2 \cdot \pi} \right)^2}{C_0}$$

(Equation 3)

where C = length between two sprockets

$p$  = chain pitch

$z_1$  = Number of teeth of sprocket 1

$z_2$  = Number of teeth of sprocket 2

Table 2: calculate the number of chain links required

Length between sprockets, in	12.3548359
Number of sprocket1	24
Number of sprocket1	11
Pitch, in	0.375
X0	83.5223919

The data is obtained from our CAD design and actual measurement. The specific raw data will be placed in the appendix for easy reference. According to the calculations in the above table, we can conclude that at least 83.5 chain links are required in the process of connecting the flywheel shaft to the drive shaft. In actual assembly, team members will adjust the number of chain links adaptively. Control the actual number of chain links between 85-95, so that we can easily connect the chain links. Slightly increasing the number of chain links can also facilitate a series of manual operations such as assembling and disassembling, adding lubricating oil, and so on.

### Finite Element Analysis

We purchased an inner shaft with a diameter of 0.5 inches, and I will perform a finite element analysis for this shaft. Since our inner axle is connected to the upper part of the human-powered vehicle and is only supported by two brackets, pressure and torsion tests are required for this part.

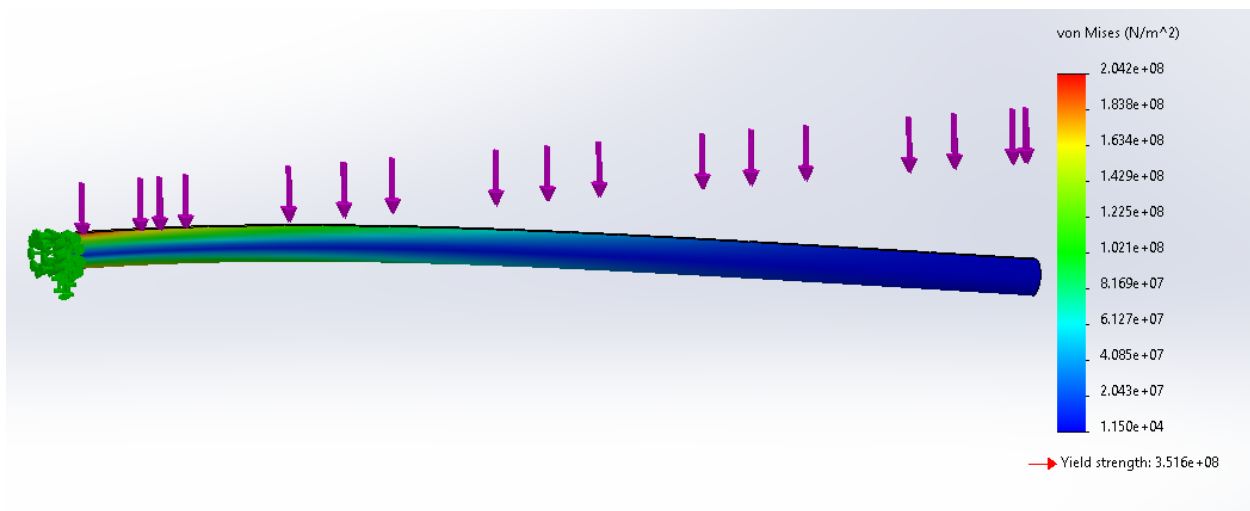


Figure 4: Stress of inner shaft

When setting constraints, I use single-sided constraints. Fix one surface of the inner shaft, and then apply a force of 200N to the key slot that is most easily deformed. Due to the design of the inner shaft, as long as the keyway can bear the weight of the flywheel and clutch, the entire shaft can bear it. When I set the force, I calculated that the weight of the flywheel and clutch is about 150N. In order to ensure the safety of the inner shaft, I set a force of 200N as the test force. According to the results of the FEA test, it can be seen from the graph that the highest stress is also lower than the Yield Strength of AISI 1020 when the force exceeds the weight of the flywheel and clutch. This shows that the inner shaft we purchased can perfectly withstand the flywheel and sprocket we designed. More FEA data will be given in the appendix.

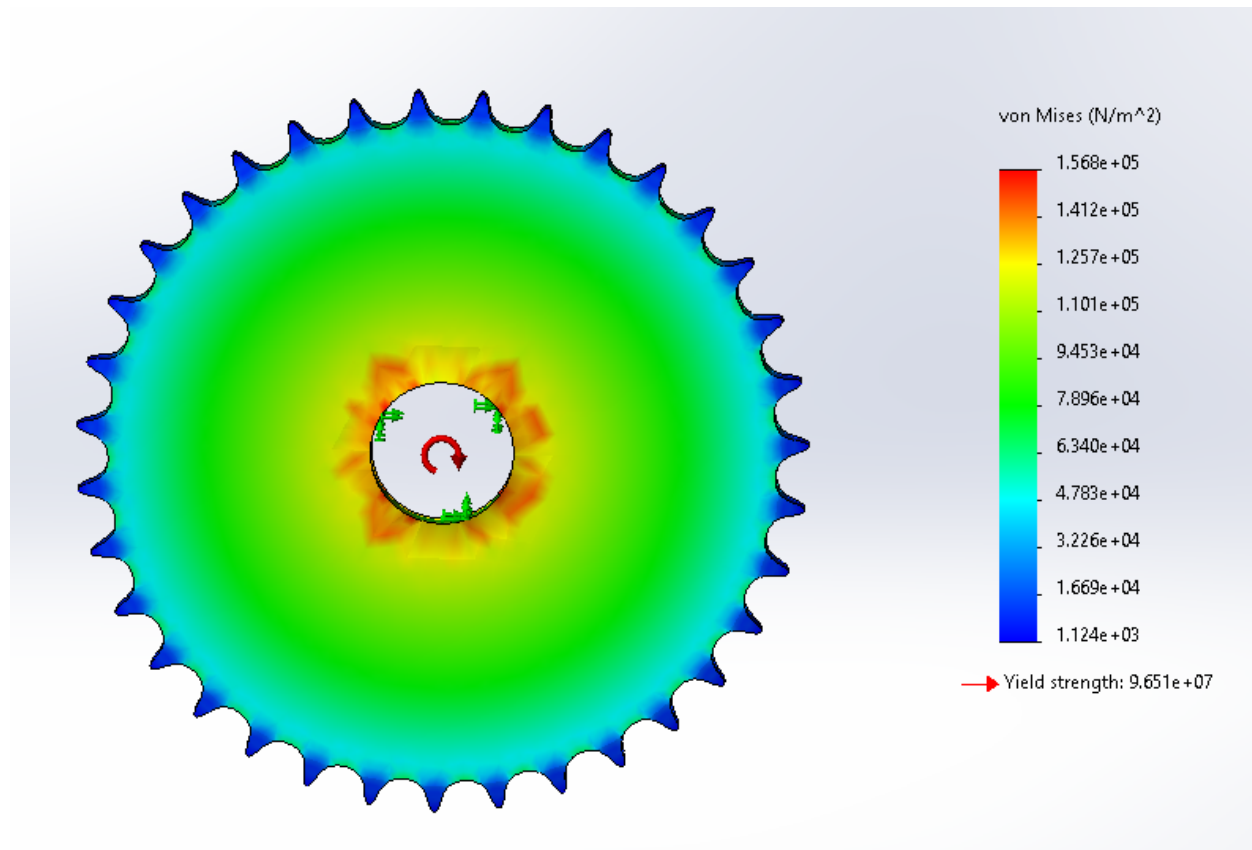


Figure 5: Stress of sprocket

The same test is performed on the sprocket, the inner diameter is set as a fixed surface, and a speed of 150 rad/s is applied. According to the calculation in Table 1, we can conclude that this speed is the speed that the sprocket cannot reach. But even in this case, a sprocket made of aluminum can still meet the requirements.

## Conclusion

This personal experiment summarized the shortcomings of the previous semester and expanded it. I combined our existing design and specifications and analyzed the sprockets and chains we need. The finite element analysis of the sprocket and the bearing sprocket shaft is carried out. In summary, our design can meet our engineering needs.

## Reference

- [1] “Sprocket Ratio Calculations”, Sciencing. 7/18/2021. [Online]. Available: <https://sciencing.com/sprocket-ratio-calculations-8043094.html>
- [2] “SPROCKET ENGINEERING DATA”, Engineering. [Online]. Available: <https://taylormhc.com/wp-content/uploads/2019/08/Sprocket-Engineering-Data.pdf>
- [3] Richard G. Budynas, J. Keith Nisbett. *Shigley's Mechanical Engineering Design Eleventh edition* , New York : McGraw-Hill Education, [2020]



## Appendix

### Appendix A: CAD model

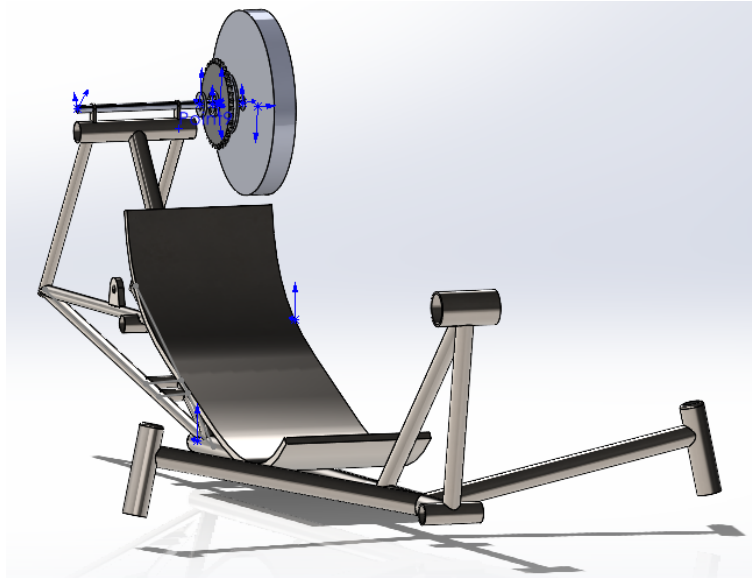


Figure A.1: General model

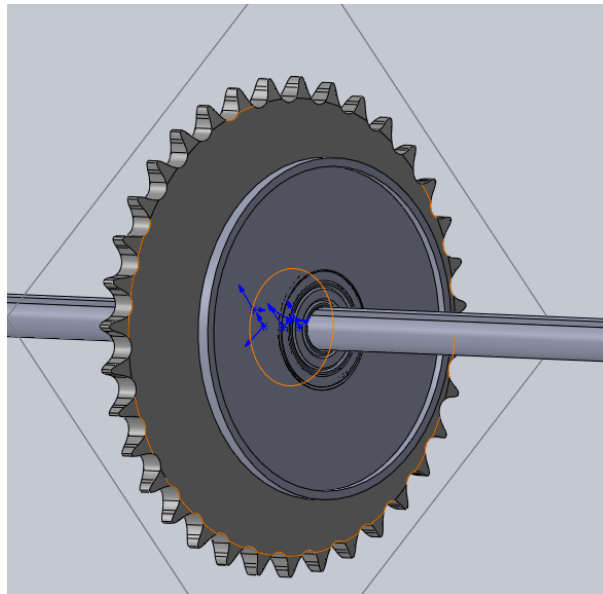


Figure A.2: Sprocket and inner shaft

## Appendix B: FEA of the inner shaft

Table B.1: Material Properties of the inner shaft

### Material Properties

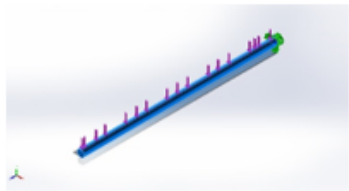
Model Reference	Properties	Components
	Name: AISI 1020 Model type: Linear Elastic Isotropic Default failure criterion: Max von Mises Stress Yield strength: 3.51571e+08 N/m <sup>2</sup> Tensile strength: 4.20507e+08 N/m <sup>2</sup> Elastic modulus: 2e+11 N/m <sup>2</sup> Poisson's ratio: 0.29 Mass density: 7,900 kg/m <sup>3</sup> Shear modulus: 7.7e+10 N/m <sup>2</sup> Thermal expansion coefficient: 1.5e-05 /Kelvin	SolidBody 1 (Boss-Extrude1)/(ME486C inner shaft 1)
Curve Data: N/A		

Table B.2: Resultant Forces of the inner shaft

### Resultant Forces

#### Reaction forces

Selection set	Units	Sum X	Sum Y	Sum Z	Resultant
Entire Model	N	-0.0247879	200.015	-0.0538635	200.015

#### Reaction Moments

Selection set	Units	Sum X	Sum Y	Sum Z	Resultant
Entire Model	N.m	0	0	0	0

#### Free body forces

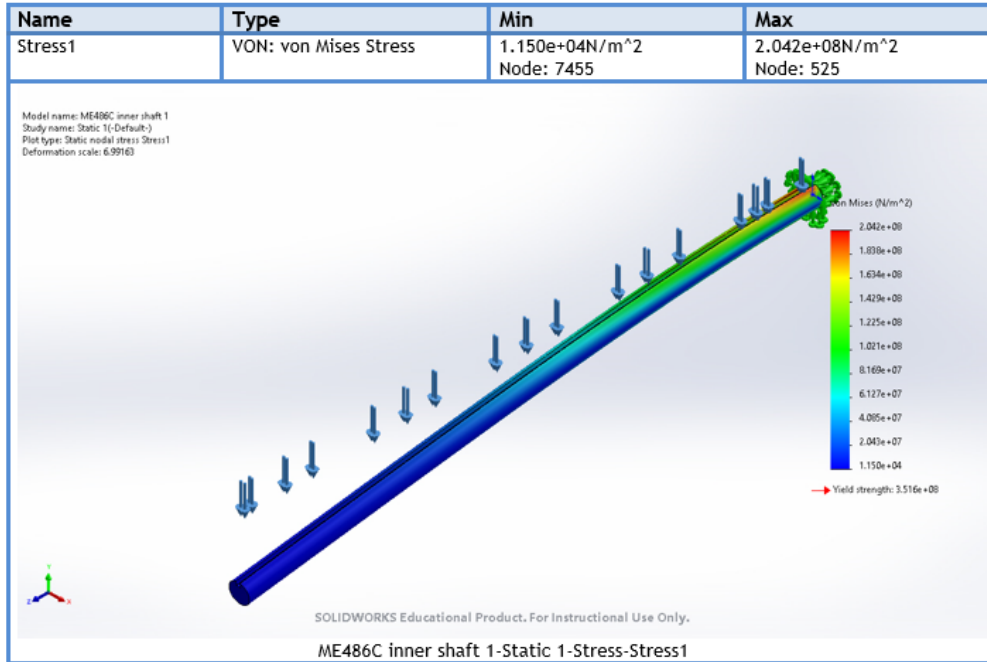
Selection set	Units	Sum X	Sum Y	Sum Z	Resultant
Entire Model	N	-0.345837	0.0100467	0.104384	0.361386

#### Free body moments

Selection set	Units	Sum X	Sum Y	Sum Z	Resultant
Entire Model	N.m	0	0	0	1e-33

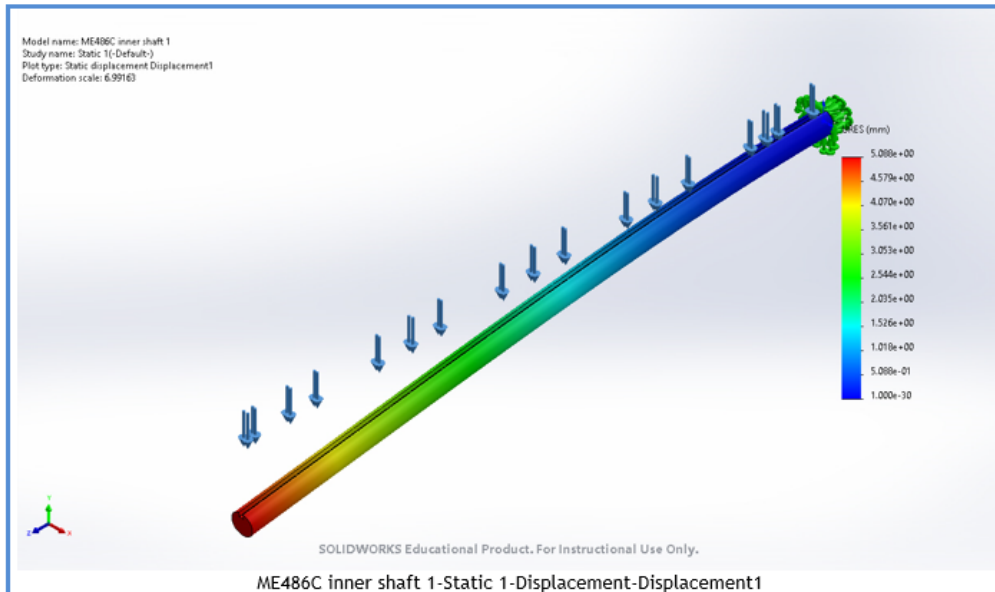
Table B.3: Stress of inner shaft

Study Results



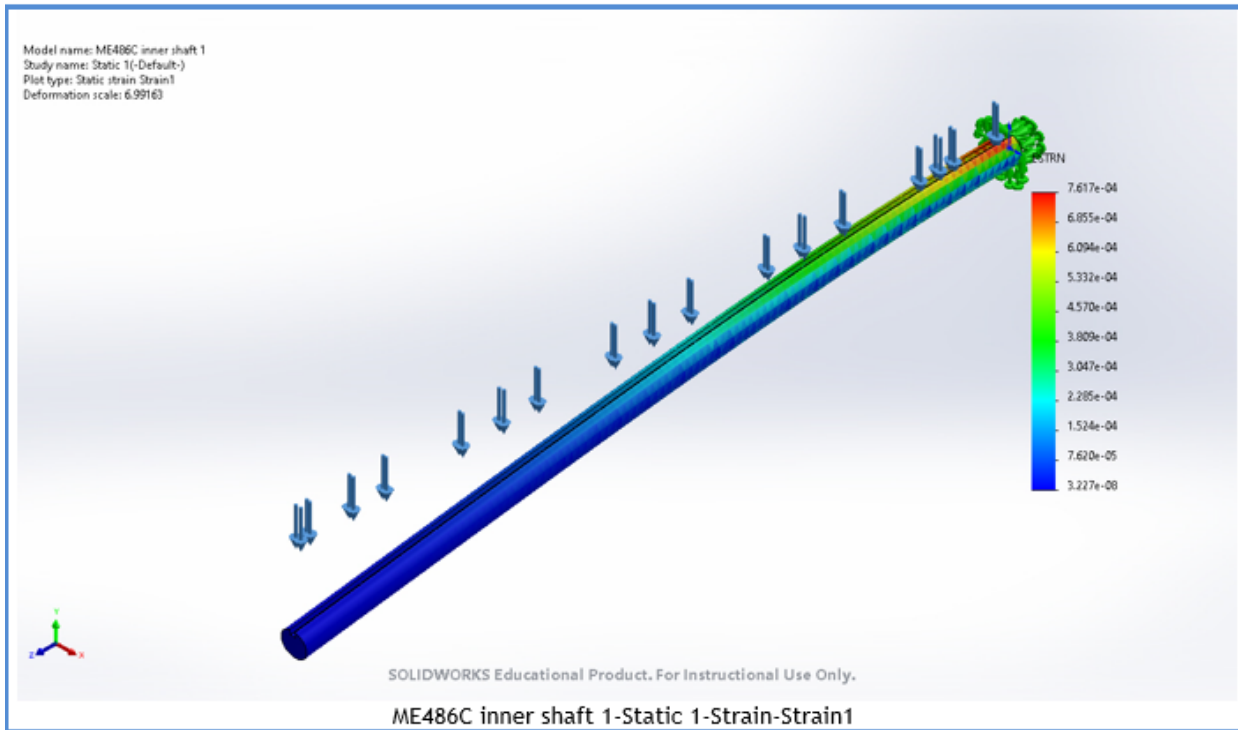
Name	Type	Min	Max
Displacement1	URES: Resultant Displacement	0.000e+00mm Node: 107	5.088e+00mm Node: 516

Table B.4: Displacement of the inner shaft



Name	Type	Min	Max
Strain1	ESTRN: Equivalent Strain	3.227e-08 Element: 3540	7.617e-04 Element: 8032

Table B.5: Strain of inner shaft



## Appendix C: FEA of sprocket

Table C.1: Material Properties of the sprocket

### Material Properties


Model Reference	Properties	Components
	<b>Name:</b> 2014 Alloy <b>Model type:</b> Linear Elastic Isotropic <b>Default failure criterion:</b> Max von Mises Stress <b>Yield strength:</b> 9.65098e+07 N/m <sup>2</sup> <b>Tensile strength:</b> 1.65445e+08 N/m <sup>2</sup> <b>Elastic modulus:</b> 7.3e+10 N/m <sup>2</sup> <b>Poisson's ratio:</b> 0.33 <b>Mass density:</b> 2,800 kg/m <sup>3</sup> <b>Shear modulus:</b> 2.8e+10 N/m <sup>2</sup> <b>Thermal expansion coefficient:</b> 2.3e-05 /Kelvin	SolidBody_1 (Cut-Extrude2) (ME486 sprocket 1)
Curve Data: N/A		

Table C.2: Resultant Forces of the sprocket

### Resultant Forces

#### Reaction forces

Selection set	Units	Sum X	Sum Y	Sum Z	Resultant
Entire Model	N	0.00466174	0.00423688	-1.93715e-07	0.00629944

#### Reaction Moments

Selection set	Units	Sum X	Sum Y	Sum Z	Resultant
Entire Model	N.m	0	0	0	0

#### Free body forces

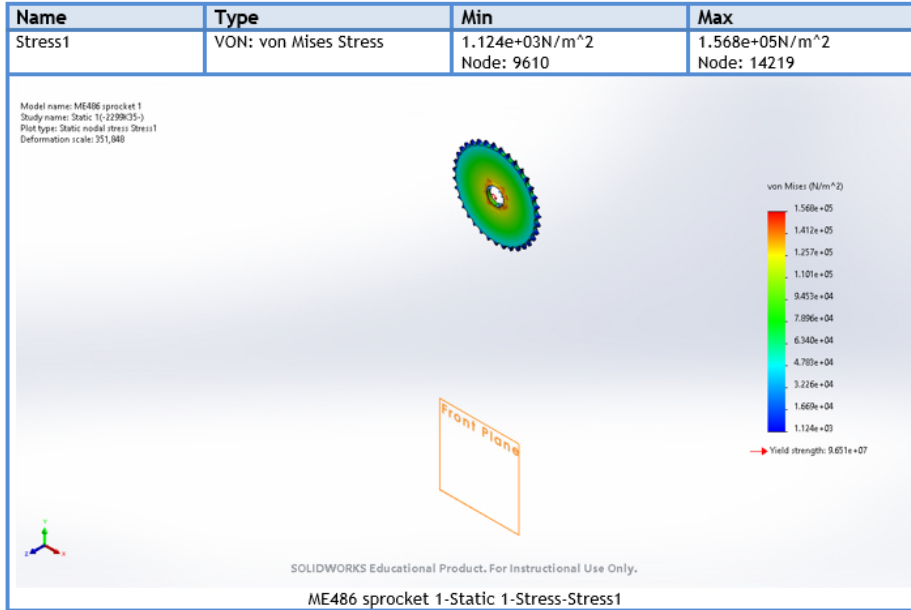
Selection set	Units	Sum X	Sum Y	Sum Z	Resultant
Entire Model	N	-0.0329967	-0.00746796	-1.93715e-07	0.0338312

#### Free body moments

Selection set	Units	Sum X	Sum Y	Sum Z	Resultant
Entire Model	N.m	0	0	0	1e-33

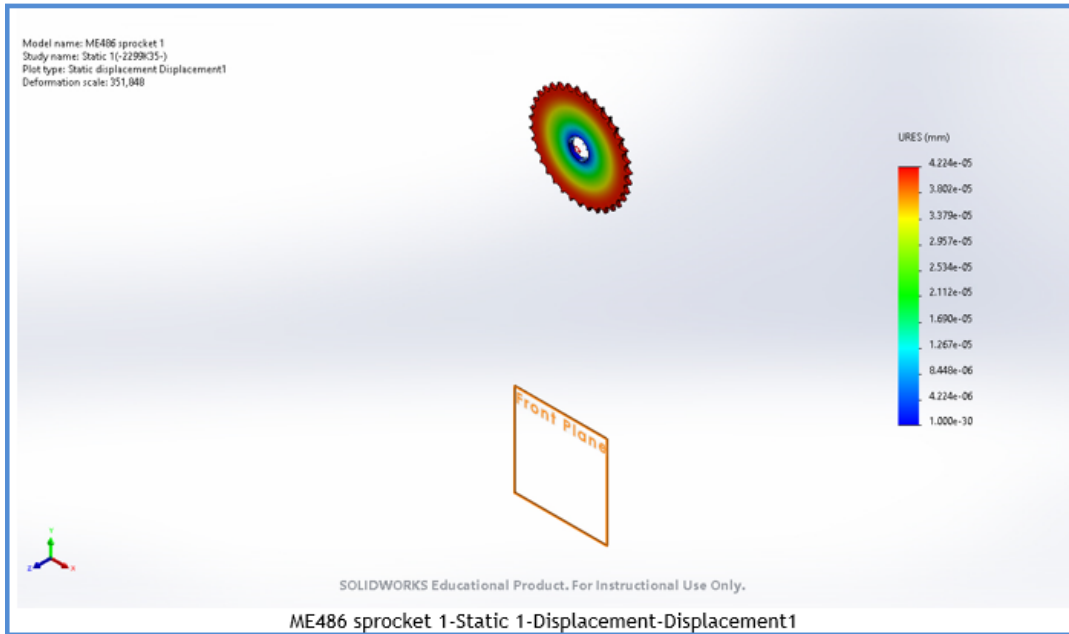
Table C.3: Stress of the sprocket

Study Results



Name	Type	Min	Max
Displacement1	URES: Resultant Displacement	0.000e+00mm Node: 1	4.224e-05mm Node: 9956

Table C.4: Displacement of the sprocket



Name	Type	Min	Max
Strain1	ESTRN: Equivalent Strain	1.293e-08 Element: 4267	1.737e-06 Element: 9895

Table C.5: Strain of the sprocket

