Bike Suspension

Final Report

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DISCLAIMER

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EXECUTIVE SUMMARY

The following final report contains the team's bike suspension results through a series of mathematical modeling. With the primary components of dampening, stress, and terrain analysis, a series of Excel code was conducted to provide mathematical equations and processes to better understand the material. Throughout the duration of the semester, the team also accumulated data to measure variables such as displacement and speed through various terrain. Brandon Lurie, the team's client, also provided important information such as proper modeling and dampening coefficients to ensure accurate results. While the dynamic of this report varies than other teams, the overall goal for this final report was to create a mathematical model to display thorough research, for the intentions are to design a motorized adjustment lever with the calculations that were found along with a data base for average consumers to use for dampening settings. Along with this, a rear floater shock was provided from a local bike shop know as '*Bike Revolution*', where the team was able to analyze the components within the shock to better understand the floater shock system. The rear shock that was donated happened to be the same that was used during the field data testing. The data collected was from Dylan's 2016 Santa Cruz Heckler. Where the rear shock is a RockShox Monarch Rt. The team was able to dismember the rear shock to determine various components to operate with fluids such as nitrogen, air, and dampening fluid. With this, there are many features that were unknow to the team as well, for the nitrogen gas was unusual concept to most consumers. With this, the terrain analysis model was conducted by determining different types of terrain, and the amount of displacement per terrain setting. The objective for this analysis was to determine which damping setting would work best for either small, medium or large bumps. Both models allow the team to have an idea as to how to approach the design portion for next semester. The goal for the design project is to use the data collected from the dampening and terrain analysis and apply it when adjusting the dampening settings on the rear floater shock. There is a large emphasis on truly understanding the mechanisms and properties of current mountain bike suspensions in order to move forward with designing a system to work alongside and optimize suspension. Throughout the report, the following customer and engineering requirements will lead to the overall process of the Black Box model, QFD, testing procedures, and design selection. Alternative software's were also used to model the desired analysis for each team member as well that also tie into the requirements. A stress analysis was also conducted through SolidWorks, Excel, and MATLAB to determine where high stress can be found when either sitting or standing on the bike frame. A pre-designed mountain bike was used from *GrabCad* where the analysis was conducted to find various Von Mises stresses throughout the geometry of the bike. An excel document was created ensure that that the MPA values were correct by using the diameters and lengths of the bike features. Also, a final MATLAB mathematical model was formed by combining both stresses at the seat post and crank arms to ensure that the frame cold hold the rider at the desired weight. And finally, a model for spring stiffness is incorporated into the dampening analysis, for both components are related in the primary mathematical model of critically dampened or under-dampened shocks under certain terrain settings. In all, the three-primary analyses met the customer needs and engineering requirements of creating a mathematical model that will allow an average mountain bike rider to determine which settings to use on the dampener for various terrains. The team's goals for next semester are to apply the model for testing with prototypes to allow a motorized device to adjust the settings on the fly without the inconvenience of getting of the bike on the trails.

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1 BACKGROUND

1.1 Introduction

Mountain biking is a sport, hobby, and even a lifestyle for those working in the industry. A key component on modern day mountain bikes is the suspension. For some mountain bikes there are only front suspension forks on the bikes giving them the name “hard tail”. However, the most versatile suspension platform is the full suspension bike. A full suspension mountain bike has a front suspension fork and a rear shock which is known to provide comfort and stability when going over harsh terrain, increased traction, and be more capable than their hard tail counterparts. Suspension on mountain bikes is meant to be taken over terrains ranging from a smooth surface to drop offs.

The best method to ensure the suspension can handle these differences in terrain is by utilizing air shocks rather than spring, or coil shocks. Air shocks have various types of adjustments that allow them to be adjusted based on the riding conditions and rider’s preference. This ability to adjust the way the bike rides, as well as weight advantage, have made air suspension the best choice for most bikes. The front fork can be locked out with a level to make the front completely rigid; the rebound can be adjusted to dictate how fast the fork reacts to bumps, and air can be added based on the weight of the rider. Rear shocks can also lock out with the damping adjustment, rebound can be increased or decreased, and air and volume spacers can be added to ensure the shock is setup properly. With all the various adjustments, adjusting suspensions can be daunting for the average consumer. Most of the time bike shops will help the rider setup the bike that will handle most terrain decent. To create the best riding bike for differing terrains, adjustments need to be made before each ride if the riding varies. For example, suspensions that are setup for jumps and drop offs will need to be adjusted if the same bike is taken to a flatter terrain with more small bumps. Without knowing exactly how each adjustment affects the bike, the average consumer could feel overwhelmed or helpless when it comes to adjusting their suspension. This project aims to define what a well setup suspension system is along with a mathematical model to follow. This then will lead the team to use a mountain bike and rear shock to apply real-world testing to validate the mathematical model. Once the mathematical model has been validated, a device will be made to help riders adjust their suspension easier. By creating a way for mountain bike suspension to be more easily adjustable, all consumers will be able to ensure their bike is perfectly dialed in no matter what terrain they are riding.

1.2 Project Description

Following is the original project description provided by the sponsor***:***
“Mountain bike suspensions are configurable based on the rider and the types of terrain where the bike is expected to be used. This project involves reverse engineering suspension systems for a mountain bike and analyzing how it would perform for varying terrains for a subset of riders using mathematical modeling. Reverse engineering a design is a common technique used to understand choices/trade-offs that a designer had to make to when developing their product as well as a starting point for creating something similar yet improved or optimized for a subset of customers. The final report should include an easy-to-use guide for selecting suspension parameters based on rider details and the range of terrains that the rider is expected to use the bike.”

The original project description involved creating a database as a reference that mountain bikers can use to fine tune their suspension. After some initial discussions with our client, Brandon Lurie, the team decided this idea would be less feasible with either too much variance in suspension platforms or products on the market that already accomplish the same goal. The only components our client wanted to keep are the idea of a mathematical model due to his knowledge in creating these models throughout his career. The client was still very open to ideas and provided the team full control of the future of the project. With this, understanding how mountain bike suspension works is pivotal to this project and the team decided on attempting to build a physical device for this project. With an updated goal in mind, the team now aims to create a mathematical model to help define a well damped suspension system and use physical devices to test the viability of the mathematical model. Once the mathematical model has been created and validated, a device will be made to allow for all riders to adjust their suspension easier.

2 REQUIREMENTS

This Senior Design Project is the capstone requirement for Mechanical Engineering majors where the team has chosen the Reverse Engineer Mountain Bike Suspension project. The team has further clarified the project to designing a system that allows for increased dynamic adjustability for a full suspension mountain bike. To accomplish this the team will utilize semester one to create a mathematical model to model and predict ideal suspension settings based on rider, bike, and terrain characteristics. From the code, a physical prototype mechanism will be developed during semester two and fitted to a to-be-determined test bike.

2.1 Customer Requirements (CRs)

The project sponsors W.L. Gore and NAU engineering, represented by Brandon Lurie and Dr. David Trevas respectively determined the baseline for customer requirements during this initial stage of project development. The team and clients together determined five customer requirements being: Base research on current mathematical models, perform extensive research on bike suspension systems, ensure the average user can utilize design, incorporate Solid Works and Excel models, and perform validation testing. These customer requirements focus on the team pursuing an analytical approach to the first semester of the design project. There is a large emphasis on truly understanding the mechanisms and properties of current mountain bike suspension in order to move forward with designing a system to work alongside and optimize suspension. This led to the team all taking on individual analysis of suspension systems such as shock quantification or linkage analysis. Each customer requirement was given a weight on a scale one to five. The Solid Works and Excel models are given a five, validation testing a four, bike suspension research a three, and user friendly and current systems research was given a two. These weights were assigned based on their correlation to gaining better understanding of the suspension system and how well they progressed the team's understanding.

The customer requirements from the preliminary report translated into similar requirements for our final proposal. The major change here is the way in which we are taking on the mathematical model. It was determined that utilizing Excel instead of MATLAB lends well to how our model has been designed. Due to this the mathematical model customer requirement is now specified as using Excel to formulate our mathematical model moving forward.

2.2 Engineering Requirements (ERs)

The engineering requirements based on the previous customer requirements can be seen below in table 1.

Table 1: Engineering Requirements with units and target value

	Engineering Requirements	Unit	Target
1	Spring and Damping Rate Critical in all Cases	lbfs/in	critical/underdamped
2	Validate Mathematical Models with other Models	%diff	+-5%
3	Test Bike Compatibility (Clamp Diameter and Geometry)	mm	28
4	Minimize Weight Addition	g	<45

5	Compact Design to Fit on Handlebars	cm^3	3
6	Durability (Material Strength)	MPa	50

The team investigated the project objective and our current hypothetical problem solution ideas to form the six engineering requirements of: Spring and damping rate critical for all cases, validate mathematical model with other models, compatibility with test bike, minimize weight addition, compact design to fit on handlebars, durability. The first requirement, spring and damping rate critical or slightly underdamped for all cases, is measured by minimal to no oscillation during the rebound motion of the suspension system and characterized by the damping coefficient. Validating mathematical model with other models is characterized by percent difference in values when comparing to Solid Works stress analysis for a specific example. The tolerance for this engineering requirement is currently hypothesized to be +- 5% however once further analysis is completed that tolerance will be set. The compatibility with test bike will come down to the future prototype being able to be fitted onto the test bike, classified by clamp diameter and mounting geometry. Tolerance for these fits will be measured with millimeters of diameter and part clearance, likely sitting around +- .5mm but subject to change. Minimizing weight addition to the bike will be targeted at sub 45 grams to limit additional weight and size. Compact design to fit on handlebars will be measured in the prototyped system's volume in cm^2 as well as clamp diameter in mm. Target volume is 3 cm^2 with a suspected tolerance of +-1 cm^2. Durability will be measured in material strength as well as impact resistance and will need to be determine through testing of current handlebar systems such as brake levers and shifters to imitate benchmarked durability. The engineering requirements will likely evolve with the project, especially during second semester when the prototyping begins. The current stage is very analytical, and research based, but tighter tolerance expectations and criteria for the project will develop alongside the move to a more physical approach to the project in semester two.

The major change in our engineering requirements affects the way we analyzed damping. It was found that the system does not need to be critical damped in all cases. A critically damped system in suspension means that the displacement of the shock returns to the original sag after encountering a bump, prior to hitting the next bump. Sag is the displacement of the shock due to rider weight on the bike at rest. In our case the rear shock has 63mm of stroke length and sag is about 16mm. An underdamped system allows the suspension the potential to return to a point less than sag displacement between bumps and therefore means that the system can be modelled to always fully rebound after encountering a bump. This change in theory for our system allows the model to target a damping coefficient between .9 and 1 and remain fully functional and safe.

2.3 Functional Decomposition

Since this project primarily focuses on a mathematical model for the current state of the project, the team could not create some diagrams that were more concept based. The Black Box Model seen below helps the team visualize how the system is currently affected by various inputs. Also, a simple process diagram helps the team maintain vision throughout the remainder of the project.

2.3.1 Black Box Model

A Black Box Model, as seen in figure 2, aims to show the inputs to the proposed system and the outputs of this system in the form of materials, energies, and signals. For this project the goal is to create a device to adjust mountain bike, MTB, suspension on the trail that is easy to understand. The material inputs will be the weight of the bike and the rider, with the rider and bike weight also being the material outputs. Material inputs can be controlled, or known, which allows the team to adjust the system based on the weight of both the person and the bike. Terrain impacts are the energy input, which causes an acceleration of the shock. This reaction to the terrain is something the team wants to be able to adjust with the device. Current adjustments on modern mountain bikes can sometimes be difficult to understand, leading to the output of a modified, or easier, way to adjust mountain bike suspension.

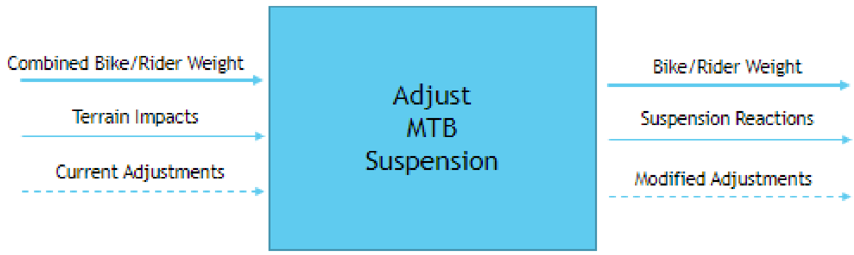

Figure 1: Black Box Model

2.3.2 Functional Model/Work-Process Diagram/Hierarchical Task Analysis

Since the preliminary report, the team has developed a process diagram that outlines the necessary major tasks throughout the project in chronological order. The process diagram below helps the team maintain vision and guides all future efforts. This models our desired progression for the project and the team is currently finishing the first box and looking to move towards the second process as the semester concludes.

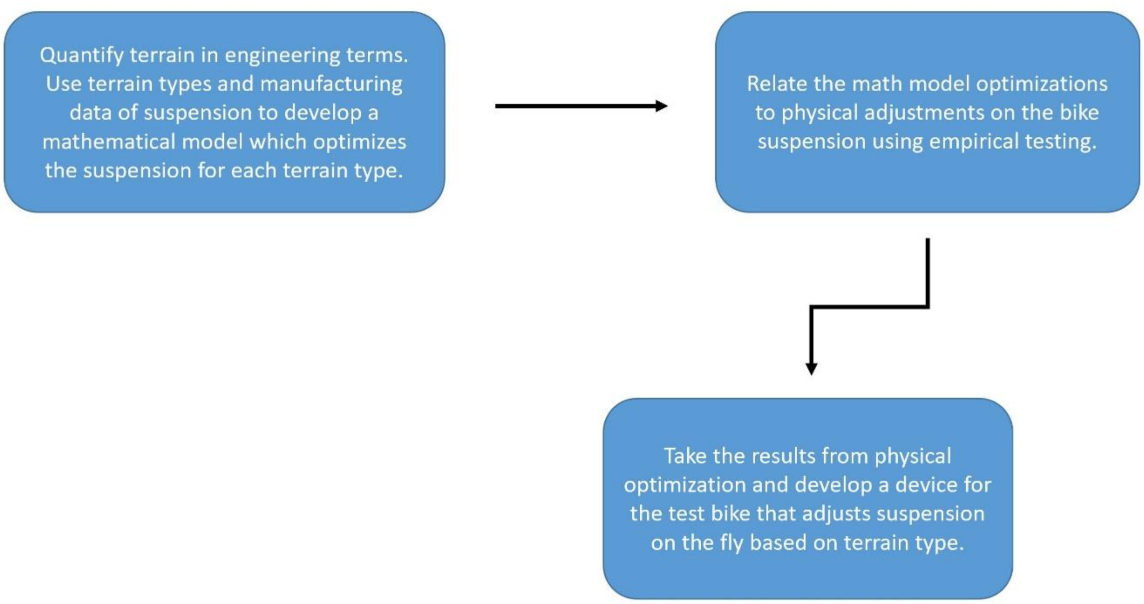

Figure 2: Process Diagram [1]

From the first bubble in the diagram above, the team has already quantified terrain types using the International Mountain Bicycling Association (IMBA) trail rating system, which may be viewed in Appendix A. This process was detailed in the preliminary report. The team has since built a mathematical model in Microsoft Excel, which will be explained in section 5. Initially, the team used MATLAB, but quickly discovered that Excel provides a more streamlined platform for spring mass dashpot calculations.

As for the other two bubbles, the team hopes to begin empirical testing over winter break. Relating math model outputs to actual bike suspension adjustments should validate the model itself. Once physical optimization data are acquired, the team will begin concept generation for the remote adjustment device. The final design should feature optimization settings for the five IMBA trail difficulties. The device should also adjust front and rear suspension independently.

These future tasks should easily fill next semester. The team will need to use this process diagram to maintain vision and stay on track as our project translates into physical testing and prototyping

2.4 House of Quality (QFD)

The House of Quality or Quality Functional Deployment (QFD) for this project ranks a small list of customer needs against the few engineering requirements developed thus far. It is subject to change and further development as the team progresses from a mathematical model to a physical mountain bike suspension adjustment device. The schematic may be viewed in Figure 3 below.

Durability						
Compact Design to Fit on Handlebars						
Minimize Weight Addition			-	++		
Compatible with Test Bike	+		++	-	-	
Validate Mathmatical Model with Other Models		++				
Spring and Damping Rate Critical for all Cases	++	+	+			+

PHASE I QFD

		Engineering Requirements (How)						Now vs Now What				
Customer Needs (What)	**Customer Weights**	Spring and Damping Rate Critical for all Cases	Validate Mathmatical Model with Other Model	Compatible with Test Bike	Minimize Weight Addition	Compact Design to Fit on Handlebars	Durability	1 Poor	2	3 Acceptable	4	5 Excellent
Base Research on Current Mathmatical Models	2	9	9							AB		
Extensive Research on Bike Suspension System	3			3			1			A	B	
Ensure the Average User can utilize Design	2	1		9	1	3	9					AB
Incorporate Solid Works and MATLAB	5		9	3	3	3					AB	
Validation Testing	4	3		9							AB	
	Absolute Technical Importance (ATI)	32	63	78	17	21	21					
	Relative Technical Importance (RTI)	14%	27%	34%	7%	9%	9%					
	Unit of Measure	Y/S	Y/S	Y/S	Kg	cm^3	lbf					
	Techical Target	Y	Y	Y		3						

Benchmarked Products	
A	Fox Live Valve
B	RockShox Brain

Figure 3: House of Quality

Engineering Requirements with the highest absolute and relative technical importance include ensuring a critical damping rate (ATI 32, RTI 14%), validating the mathematical model with empirical testing of a physical bike (ATI 63, RTI 27%), and making sure the suspension adjustment device developed by the team is compatible with the bike tested (ATI 78, RTI 34%). The critical damping rate will vary as terrain and trail difficulty varies, and therefore is unknown at this point. Hopefully the team will confirm critical damping rates calculated in the mathematical model when physically testing the mountain bike, validating the model. At the end of the project, the team should have a suspension adjustment device designed specifically for the bike tested. Any rider should be able to adjust the suspension to the desired terrain setting based on suggestions from the previously developed mathematical model. This QFD helps the team realize the most important engineering requirements in order to reach the end goal.

2.5 Standards, Codes, and Regulations

ASTM has standard for mountain bikes in the following paragraph:

> "This specification establishes the performance requirements and associated test methods for qualifying designs of suspension and nonsuspension production forks employed on bicycles that are intended for use in Condition 3 topography. This kind of condition pertains to rough trails, rough unpaved roads, and rough technical areas and unimproved trails, wherein contact with the irregular terrain and momentary loss of tire contact with the ground may occur during usage. The forks shall go through compression load, bending load, impact resistance, and fatigue tests. Models that fail to meet the specified test requirements shall be rejected." [2]

There are no specific regulations that abstracts the usage of new types of damper fluid, or how much nitrogen is used in the shock, or the usage of any material specifically. As can be summarizes from the ASTM paragraph, if the part works properly and effectively works as intended, it won't get rejected [2]. Also, flammable gas like nitrogen is to be used, manufacturers are required to seal it preventing any leakage [3]. This will not affect our project in any major ways since we are not manufacturing our own suspension, instead we are going to use an existing suspension that already met all the requirements.

Table 1: Some Standards that are frequently used

Part	**Standard**
Fork Material	Steel, Aluminum, Carbon Fiber, and Titanium. [4]
Travel Design	80mm-180mm+ [5]
Optimal Stroke	37.5mm-72.5mm [5]

These are not set-in-stone by any means, they are only frequently used in bikes. This results in an extremely wide range of possibilities. The part that proved to provide the most possibilities was the damper fluid. There are more than 100 known damping fluids that manufacturers could use in bikes dampers [6].

3 Testing Procedures (TPs)

Several tests are essential to this project. A static suspension test is needed to determine initial values for the mathematical model. Next, dynamic testing will be used to validate the mathematical model and acquire optimized adjustment data. Finally, prototype testing will be required to prove the viability of the final device design.

3.1 Testing Procedure 1: Sag Test

3.1.1 Testing Procedure 1: Objective

This first test method measures Sag. Sag is the suspension displacement when the rider first gets on the bike. Acceptable values are 20-30% of shock stroke length. The sag test determines initial displacement values for the mathematical model. These values influence the displacement profile over time when terrain impacts act on the suspension.

Figure 4: Dylan performing some tests after setting sag

3.1.2 Testing Procedure 1: Resources Required

For this test, the team chose a member's personal mountain bike, a 2016 Santa Cruz Heckler. The bike features the least progressive suspension geometry and uses RockShox air suspension components. An air shock pump was needed to adjust air pressure to rider weight specifications, and a metric measuring tape was used to measure shock stroke length and sag.

3.1.3 Testing Procedure 1: Schedule

The team performed a level sag test two weeks ago. After air pressure was optimized for team member Dylan's weight, an average sag value from three measurements was calculated. The test required 20 minutes to complete. In the future, however, the team will need to perform sag tests at grades up to 20%. Monday afternoons work best for the team, and these additional tests should not require more than an hour

3.2 Testing Procedure 2: Dynamic Model Validation

4.2.1 Testing Procedure 2: Objective

This second test type involves dynamic suspension displacement measurements using a measuring tape. If possible, the team will also use Arduino sensors to measure time increments during compression and rebound. This test must encompass each terrain type quantified in the IMBA trail rating system.

Dynamic testing will help relate mathematical model outputs to physical suspension adjustments and thus validate the model.

4.2.2 Testing Procedure 2: Resources Required

In this test, the same Santa Cruz Heckler, shock pump, and metric tape measure will be used, with the addition of an Arduino mega board, appropriate sensor, and PC for data analysis. The team will also need to find trails that follow the IMBA trail rating system.

4.2.3 Testing Procedure 2: Schedule

The team has already begun this stage of testing, generally meeting on Mondays for 2 hours or so. Completion will most likely require 20-30 more hours of group effort spread out over the next two months.

3.3 Testing Procedure 3: Device Testing

3.3.1 Testing Procedure 3: Objective

Prototype testing will prove the viability of the adjustment device design. The team needs to make sure it is robust enough to handle harsh trails, and functional enough to validate the mathematical model.

3.3.2 Testing Procedure 3: Resources Required

This test will require everything mentioned in the previous tests as well as the adjustment device itself. The device will be mounted to the test bike and undergo dynamic testing similar to test 2.

3.3.3 Testing Procedure 3: Schedule

Ideally, the team will reach this testing stage by March of next year. The usual Monday meetings should suffice. Once validation is complete, the team will be ready to present the mathematical model and device as a cohesive design.

4 DESIGN SELECTED – First Semester

The final mathematical model will include input variables such as terrain, rider size and various shock dimensions, then output the behavior of the shock which will give the team insight on what adjustments to recommend to the rider. Microsoft excel will be used to create the mathematical model with a sheet for each level of terrain. An analysis of the spring stiffness will be included in this section since air shocks do not behave as linear springs. Once the spring stiffness is estimated using the linear correlation of the data, the data will be compiled and analyzed. While the mathematical model is finished, the team still needs to analyze how outputs from the model can be converted into suspension adjustments.

4.1 Design Description

4.1.1 Spring Stiffness

Since most modern mountain bikes use air shocks instead of a spring or coil shock, a model needed to be generated to include various parameters of the shock's dimensions and the amount of pressure inside the shock. The equation (equation 1 below) used to model the air shocks on mountain bikes spans from U.S. Patent No. 4,629,170, which is then discussed in an analysis in European Patent No. 0285726B1.

$$K = \frac{nPA_e^2}{V} + \frac{P_g dA_e}{dH} \tag{1}$$

In this equation, K is the stiffness of the spring, P is the absolute internal pressure, Pg is the gauge pressure, Ae is the effective area acted on by air pressure, V is the air volume, n is the ratio of specific heats, and   is the change in effective area with spring height. The absolute internal pressure will vary by the rider's weight and the type of shock. Area and volume will also be dependent on the specific shock and can be adjusted for any shock if the dimensions are known. A graph of the spring stiffness versus the length of the shock can be seen below in figure 4.

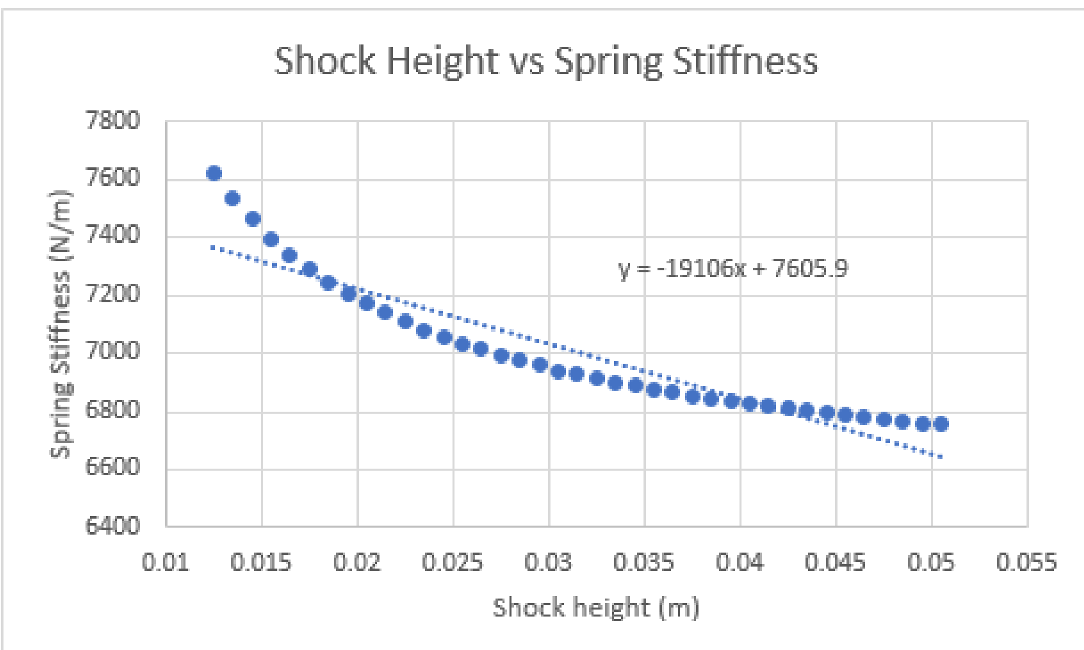

Figure 5: Spring stiffness through length of shock stroke

This graph shows how the shock stiffness begins to decrease as the rider gets into the travel. For the analysis of the suspension, the team is using the spring-mass dashpot (a differential equation) that needs to be solved numerically. To help simplify the analysis, a linear trend line will be utilized, and stiffness values will be found using the equation of the line. Spring stiffness will vary based on the difficulty of the terrain. For example, a rider on an easier trail will not experience as many, or as large, of bumps meaning they will not use as much of their travel. Versus someone who is going off of a 5 foot drop off and bottoming out their shock using the entire length of the shock. This will be incorporated in the final mathematical below.

4.1.2 Stress Analysis

The following MATLAB code indicates the stress levels on the crankarms and the seat post (sitting and standing while riding). A SolidWorks Simulation was conducted with a pre-designed mountain bike from *Grab CAD* to determine the Von Mises stresses (2) throughout the geometry of the bike. As seen in the SolidWorks figure (5), the color coordinated bike frame indicates where small, medium, and high stress levels occur. The MATLAB code (6) is an additional source to back up the original trial on the SolidWorks simulation. The results show that when sitting on the seat at an applied load of 1,500N will result in a Von Mises stress of 2.6Mpa and the crankarm will have a Von Mises stress of 3.4Mpa. The MATLAB code is the skeleton of the SolidWorks process, for this information is not seen in the simulation. This allows the team to better understand the mathematical modeling with the stress analysis when analyzing stress throughout the bike frame. Overall, between both software's, the same data was found, allowing the team to know the final Von Mises stresses throughout the geometry of the bike.

$$\sigma_v = \sqrt{\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2}$$

Equation 2: Von Mises Stress

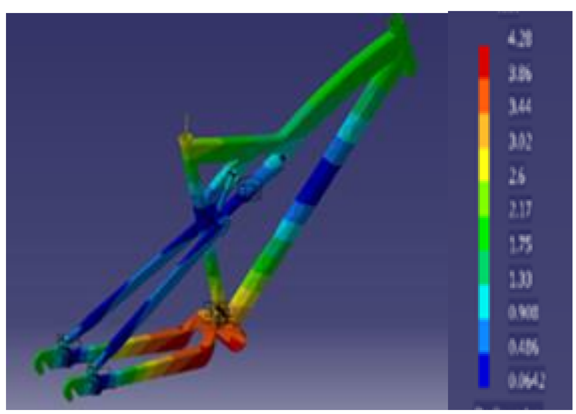

Figure 6: SolidWorks Von Mises Stress

```
% Jacob Cryder
% Stress Analysis
% 10/22/2020

% Givens
F = 1521 % Applied Load in Newtons
L1 = .175 % Length of Crank arm in Meters
L2 = .126 % Cneter of pedal to  crank arm in Meters
D = 12 % Diameter of Crank Arm in mm
Sy = 310 % Tesile Strength
BendingMoment = (F*L1) %In N*m
TorsionalMoment = (F*L2) %In N*m

TensileStresss = (32*BendingMoment)/(pi*D^(3))
ShearStress = (16*TorsionalMoment)/(pi*D^(3))

MaxStress = (TensileStresss/2) + sqrt((TensileStresss/2)^(2)+ShearStress^(2)) % In Mpa
MinStress = (TensileStresss/2) - sqrt((TensileStresss/2)^(2)+ShearStress^(2)) % In Mpa

VonMises = (MaxStress^(2)+MinStress^(2)-(MaxStress*MinStress));

fprintf ('VonMises = %d\n MPA',VonMises)

FactorofSafety = Sy/MaxStress
```

```
F = 1521
L1 = 0.1750
L2 = 0.1260
D = 12
Sy = 310
BendingMoment = 266.1750
TorsionalMoment = 191.6460

TensileStresss = 1.5690
ShearStress = 0.5648

MaxStress = 1.7512
MinStress = -0.1822

VonMises = 3.418904e+00
 MPA
FactorofSafety = 177.0225
```

Figure 7: MATLAB Code for stress on Crankarms

4.1.3 Mathematical Model

For this semester, the desired product was a mathematical model that outputs suspension reactions to various terrain types. Initially, the team thought MATLAB was the best platform, but Microsoft Excel turned out to be more streamlined. In figure 8 below, all inputs are applied to the front air fork and rear air spring independently. This is important because the components behave differently due to frame geometry and front/rear mass bias. The green cells are user defined, and yellow cells calculate initial values based on the user defined inputs. Bike mass and rider mass are transformed into front and rear mass bias. Spring stiffness comes from the stiffness analysis above, and the damper value represents the mass flow rate of oil inside the damper. The leverage ratio influences rear shock characteristics relative to the front fork, therefore, initial shock speed will always be less for the rear shock. Stroke length is also less than half that of the front fork, causing the displacement profile to look shorter.

Initial Values			
Bike Mass	bm	13.5	kg
Person Mass	pm	84	kg
Overall Mass	m	97.5	kg
Front sprung mass	mf	32.5	kg
Rear sprung mass	mr	65	kg
Front fork stiffness	kF	2000	N/m
Rear spring stiffness	kR	7400	N/m
Front damper value	λF	500	kg/s
Rear damper value	λR	1360	kg/s
Front damping coefficient	ςF	0.9806	Ns/m
Rear damping coefficient	ςR	0.9805	Ns/m
Front natural frequency	Fωn	7.8446	1/s
Rear natural frequency	Rωn	10.6699	1/s
Front Initial shock speed	v0	2.6	m/s
Front sag	x0f	0.035	m
Rear sag	x0r	0.016	m
Rear initial shock speed	v0r	1.0924	m/s
Front damped frequency	Fωd	1.5385	1/s
Rear damped frequency	Rωd	2.0982	1/s
Front Fork Travel	FT	0.15	m
Rear Shock Travel	RT	0.063	m
Leverage Ratio	LR	2.38	unitless

Figure 8: Math Model Inputs [7]

These inputs run through an if statement that determines the damped situation (underdamped, critically damped, or overdamped) and then uses the appropriate equation to output a graph of displacement vs time for front and rear suspension independently (Figure 9). The oscillations both near the baseline within one second reliably, and the interval between data points is 0.01s. Given the inputs above, front fork displacement nearly reaches maximum stroke length, and the rear shock reaches 80% of its stroke length.

Critically damped and slightly underdamped situations are desirable for most terrains. Overdamped situations do not allow the shock to sufficiently rebound before the next impact, creating a harsh sensation. The equations behind critically damped and underdamped situations are listed respectively below.

$$x(t) = [x_0 + (v_0 + \omega_n x_0)t]e^{-\omega_n t} \quad \text{Equation 3}$$

$$x(t) = e^{-\varsigma\omega_n t}[x_0 cos\omega_d t + \frac{v_0 + \varsigma\omega_n x_0}{\omega_d} sin\omega_d t] \quad \text{Equation 4}$$

$$\omega_d = \omega_n\sqrt{1-\varsigma^2} \quad \text{Equation 5}$$

Damped natural frequency ω_d is a function of natural frequency ω_n and ς in the underdamped equation.

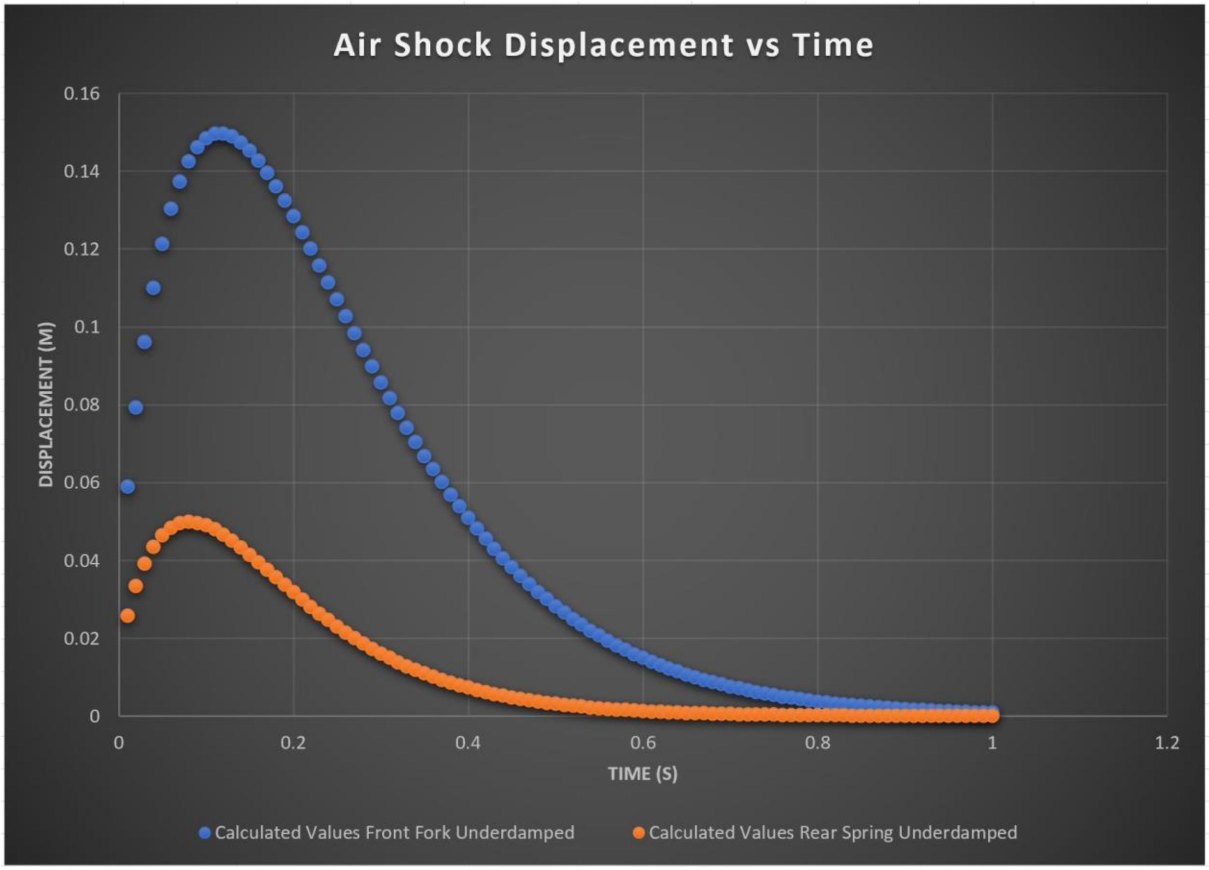

Figure 9: Math Model Outputs [7]

These results reflect level terrain inputs. The model still needs grade and uphill/downhill inputs to be built in. Those should be completed soon.

4.1.4 Final Design

The final device design has yet to undergo development. The team must first validate the mathematical model and perform the necessary concept generation/design selection processes.

4.2 Implementation Plan

This mathematical model will be implemented with real world testing of the bike and studying how different adjustments affect the ride. Once some experimental data is collected the team will be able to analyze the effectiveness of the model and then optimize the suspension for all terrains. To optimize the suspension settings, the team is looking for a company or local bike shop to allocate a bike for the team to test. If not, the Santa Cruz Heckler used for the sag test will be adequate. During these tests, the team will implement an Arduino with proximity sensors to measure how the bike suspension behaves on various terrains. Ideally, the team will build an alpha prototype (and possibly a beta) of the final adjustment device design. Materials and costs cannot be determined at this time, but the device should stay well within budget, especially if the team uses Dylan's Santa Cruz as the test bike.

5 CONCLUSIONS

This semester has seen various changes to this project from the initial project proposal to the direction the team is looking to pursue for next semester. Originally, the team was tasked with creating a database for mountain bike riders to help adjust their suspension; now the team is looking to create a mathematical model to optimize suspension for various terrain type, relate these outputs to adjustments on the bike, then create a device to enable for riders to adjust their suspension on the trail. Our client was very open to ideas for project direction and was willing to shift the project into something the team wanted to do. The client's main requirements were for the project to include a mathematical model and be based around bike suspension in some way. Beginning this semester, the team focused more on understanding mountain bike suspension through in-depth research of specific topics. Austin, Jake, and Dylan investigated stresses on the bike, geometry, and forces the bike may experience. Tyson, Erik, and Suliman researched mathematical equations and how these could be related to a mathematical model. The team then put the individual analysis together and started working towards the common goal of creating a mathematical model that can handle various terrain, rider, and bike inputs then output the optimal values to ensure the shock is slightly under damped or critically damped. Each of the individual tasks help the team to create a more complete mathematical model that could be implemented and tested with real world adjustments on the bike itself. The mathematical model created includes a varying spring stiffness since the team is studying air shocks and uses the spring mass dashpot equation to find if the shock is damped correctly. Graphs of the spring stiffness and shock displacement over time are shown to help analyze how the shocks behave. This model is almost entirely completed, and the next phase of the project is to start testing the model, both with empirical testing and using Excel to optimize the function. Moving forward, the team hopes to build a device, with the help of the mathematical model, to ensure the rider is always at the best suspension setting for their terrain. This device will allow for riders to adjust their settings on the trail without having to take their focus off of the journey ahead of them.

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7 APPENDICES

7.1 Appendix A: IMBA Trail Rating System [8]

IMBA Trail Difficulty Rating System

	EASIEST WHITE CIRCLE	EASY GREEN CIRCLE	MORE DIFFICULT BLUE SQUARE	VERY DIFFICULT BLACK DIAMOND	EXTREMELY DIFFICULT DBL. BLACK DIAMOND
TRAIL WIDTH	72" (1,800 mm) or more	36" (900 mm) or more	24" (600 mm) or more	12" (300 mm) or more	6" (150 mm) or more
TREAD SURFACE	Hardened or surfaced	Firm and stable	Mostly stable with some variability	Widely variable	Widely variable and unpredictable
AVERAGE TRAIL GRADE	Less than 5%	5% or less	10% or less	15% or less	20% or more
MAXIMUM TRAIL GRADE	Max 10%	Max 15%	Max 15% or greater	Max 15% or greater	Max 15% or greater
NATURAL OBSTACLES AND TECHNICAL TRAIL FEATURES (TTF)	None	Unavoidable obstacles 2" (50 mm) tall or less Avoidable obstacles may be present Unavoidable bridges 36" (900 mm) or wider	Unavoidable obstacles 8" (200 mm) tall or less Avoidable obstacles may be present Unavoidable bridges 24" (600 mm) or wider TTF's 24" (600 mm) high or less, width of deck is greater than 1/2 the height	Unavoidable obstacles 15" (380 mm) tall or less Avoidable obstacles may be present May include loose rocks Unavoidable bridges 24" (600 mm) or wider TTF's 48" (1,200 mm) high or less, width of deck is less than 1/2 the height Short sections may exceed criteria	Unavoidable obstacles 15" (380 mm) tall or less Avoidable obstacles may be present May include loose rocks Unavoidable bridges 24" (600 mm) or narrower TTF's 48" (1,200 mm) high or greater, width of deck is unpredictable Many sections may exceed criteria