Red Feather Solar Furnace

Individual Analytical Analysis

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2020

Mechanical Engineering

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Introduction

One of the most important aspects of this solar furnace design is the overall design of the casing. The way it is designed will impact how long it takes for the system to heat up. Heat loss through the boundaries of the system is inevitable, but utilizing a design that minimizes this, increases the functionality of the device. During the concept generation and selection portion of the design, multiple casing geometries were developed. After evaluating them in a decision matrix, the three designs seen in figure 1 below scored the highest. In the decision matrix in appendix A, the systems below are designs 1, 2 and 3 respectively.

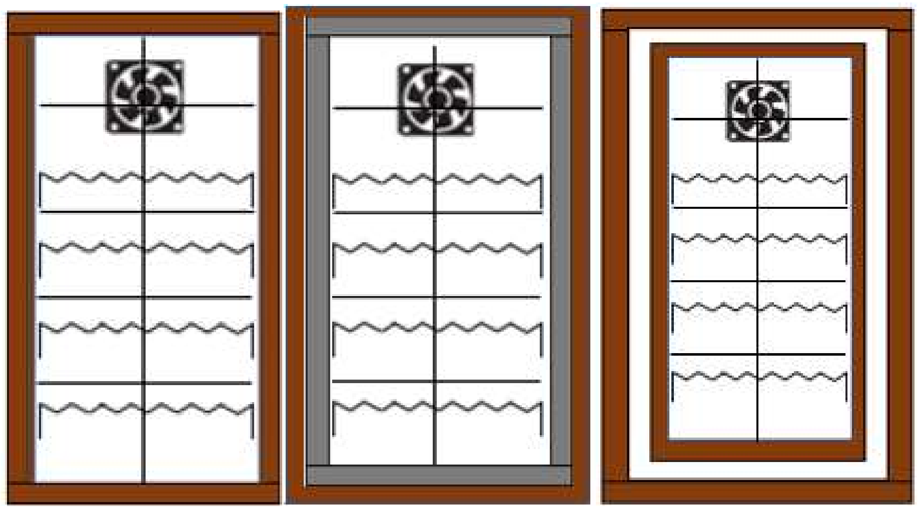


Figure 1: Top three solar furnace casing options.

The scoring of the decision matrix did not take into account actual calculations, but rather a general understanding of the thermal conductivity of the materials to be used. The following calculations are one-dimensional computations of the heat transfer out of the sides of the device for each of the three geometries above. The heat loss out of the back of the device would also be calculated however, all three systems have the same aluminum and wood design on the back, so the value for this would be the same. Technically, the heat loss will change depending on the current temperature of the device, but in order to determine the best design it is only necessary to compare the heat loss at a single temperature difference. The maximum expectation of the device is a temperature gain of 50°C above ambient. Evaluating the heat loss at the maximum temperature will provide the maximum heat loss value.

Calculations/Results

There are a few calculations that go into calculating the heat transfer out of a system. The heat transfer, q is defined by equation 1 below. In that equation, ΔT is the temperature system across the boundaries, and $\sum R_{Th}$ is the sum of the thermal resistances. There are two types of thermal resistances, one for conductive heat transfer (equation 2) and one for convective heat transfer (equation 3). In conductive heat transfer, L is the length/thickness of the wall, k is the thermal conductivity, and A is the cross-sectional area of the wall. In convective heat transfer, h is the heat transfer coefficient, and A is the cross-sectional area of the wall.

$$q = \frac{\Delta T}{\sum R_{Th}}$$ Equation 1 [1]

$R_{conduction} = \frac{L}{kA}$ Equation 2 [1]

$R_{convection} = \frac{1}{hA}$ Equation 3 [1]

The first design is a simple wooden boundary. This design is being considered due to its simplicity and affordability. The thermal conductivity of balsa wood is approximately 0.048 W/mK [2]. Below in figure 2 is the simple resistive network associated with this design, the calculation of those thermal resistances, and the calculation of the heat loss in Watts. This design results in a heat loss of 34.01W.

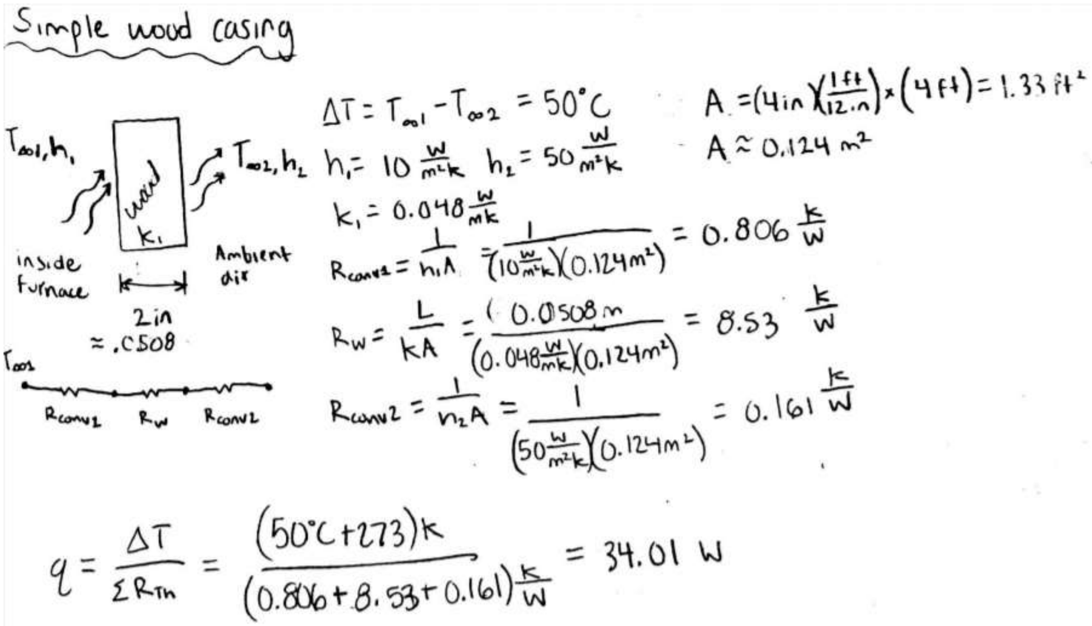
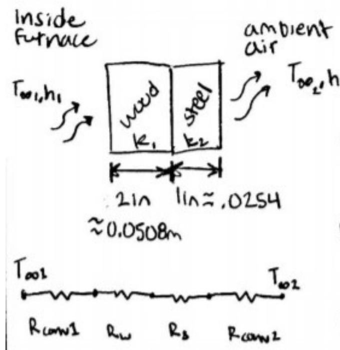


Figure 2: heat loss calculation for simple wooden casing.

The second design is the simple wooden boundary with a layer of steel. The idea behind this design is that the steel will absorb heat better than the wood on the vertical sides of the device, but the wood will prevent too much heat loss through the boundaries. The thermal conductivity of steel is approximately 54 W/mK [2]. Below in figure 3 is the simple resistive network associated with this design, the calculation of those thermal resistances, and the calculation of the heat loss in Watts. This design results in a heat loss of 33.99W.

Wood/Metal Casing



$$\Delta T = T_{\infty 1} - T_{\infty 2} = 50°C \qquad A = (4in)\left(\frac{1ft}{12in}\right) \times (4ft) = 1.33ft^2$$

$$h_1 = 10 \frac{W}{m^2K} \qquad k_1 = 0.048 \frac{W}{mK} \qquad A \approx 0.124m^2$$

$$h_2 = 50 \frac{W}{m^2K} \qquad k_2 = 54 \frac{W}{mK}$$

$$R_{conv1} = \frac{1}{h_1 A} = \frac{1}{\left(10 \frac{W}{m^2K}\right)(0.124m^2)} = 0.806 \frac{K}{W}$$

$$R_W = \frac{L}{k_1 A} = \frac{0.0508m}{\left(0.048 \frac{W}{mK}\right)(0.124m^2)} = 8.53 \frac{K}{W}$$

$$R_{st} = \frac{L}{k_2 A} = \frac{0.0254m}{\left(54 \frac{W}{mK}\right)(0.124m^2)} = 0.00379 \frac{K}{W}$$

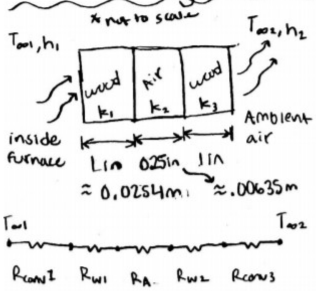
$$R_{conv2} = \frac{1}{h_2 A} = \frac{1}{\left(50 \frac{W}{m^2K}\right)(0.124m^2)} = 0.161 \frac{K}{W}$$

$$q = \frac{\Delta T}{\Sigma R_{Th}} = \frac{(50°C + 273)K}{(0.806 + 8.53 + 0.00379 + 0.161)\frac{K}{W}} = 33.99 W$$

Figure 3: heat loss calculation for wooden casing with metal inner layer.

The third design is the simple wooden boundary with an air gap, and then another layer of wood. The idea behind this design is that the air will be a cheap way to insulate the boundaries, further reducing heat loss. The thermal conductivity of air is approximately .025 W/mK [2]. Below in figure 4 is the simple resistive network associated with this design, the calculation of those thermal resistances, and the calculation of the heat loss in Watts. This design results in a heat loss of 27.95W.

Wood Casing w/ Air Gap



$$\Delta T = T_{\infty 1} - T_{\infty 2} = 50°C \qquad A = (4in)\left(\frac{1ft}{12in}\right)\times(4ft) = 1.33\,ft^2 \approx 0.124m^2$$

$$h_1 = 10\frac{W}{m^2K} \quad h_2 = 50\frac{W}{m^2K}$$

$$k_1 = 0.048\frac{W}{mK} \quad k_2 = 0.025\frac{W}{mK} \quad k_3 = 0.048\frac{W}{mK}$$

$$R_{conv1} = \frac{1}{h_1 A} = \frac{1}{(10\frac{W}{m^2K})(0.124m^2)} = 0.806\frac{K}{W}$$

$$R_{w1} = \frac{L}{k_1 A} = \frac{0.0254m}{(0.048\frac{W}{mK})(0.124m^2)} = 4.27\frac{K}{W}$$

$$R_A = \frac{L}{k_2 A} = \frac{0.00635m}{(0.025\frac{W}{mK})(0.124m^2)} = 2.05\frac{K}{W}$$

$$R_{w2} = \frac{L}{k_3 A} = \frac{0.0254m}{(0.048\frac{W}{mK})(0.124m^2)} = 4.27\frac{K}{W}$$

$$R_{conv2} = \frac{1}{h_2 A} = \frac{1}{(50\frac{W}{m^2K})(0.124m^2)} = 0.161\frac{K}{W}$$

$$q = \frac{\Delta T}{\Sigma R_{Th}} = \frac{(50°C + 273)K}{(0.806 + 4.27 + 2.05 + 4.27 + 0.161)\frac{K}{W}} = 27.95W$$

Figure 4: heat loss calculation for wooden casing with air gap.

Conclusion

As observed in the figures above, the casing design with the lowest heat loss is the wood casing with the air gap. This result was expected, and is the reason that design scored the highest on the decision matrix (appendix A). The air gap design results in a 27.95 Watt heat loss at max temperature. Both of the other designs result in approximately 34 Watts of heat loss. This is a significant improvement however, the higher efficiency needs to be weighed against the complexity of adding the air gap. The heat loss could be reduced further by increasing the width of the gap as well as the thickness of the sidewalls although that leads to issues with weight and overall size. These calculations will be taken into consideration when working towards the final system prototype. These calculations do however narrow down the options to either the simple wood casing, or the wood casing with an air gap since the steel layer did little to reduce heat loss.

References

[1] T. L. Bergman, A. Lavine, and F. P. Incropera, *Fundamentals of heat and mass transfer*. Hoboken, NJ: John Wiley & Sons, Inc., 2019.

[2] Thermal Conductivity of Metals, Metallic Elements and Alloys. Engineeringtoolbox.com, 2020. [Online] Available: https://www.engineeringtoolbox.com/thermal-conductivity-metals-d_858.html [Accessed 13 March 2020].

Appendix A

Table A-1: Decision matrix.

Redesign																					
Criteria	**Weight**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
Cost	**0.35**	3	2	3.5	2	3	2	4	3	3.5	4	3	2.5	2.5	4	3	3	3	4.5	4	5
Heat Gen	**0.35**	4.5	4.5	4	4	3.5	3.5	3	3	2.5	2.5	2.5	3	3.5	2	3	3.5	2	2.25	2.5	1.5
Manufacturability	**0.15**	4	4	4.25	3	3.5	3	4	4	4	3.5	3.5	3	2.5	4	3	4	4	3.5	3.5	4
Durability	**0.15**	4	4.5	4	4.5	4	3.5	3	3	4.5	3	3	2	3	2.5	2	2	4	4	4	2
Sum (out of 5)	1	**3.83**	**3.55**	**3.86**	**3.23**	**3.40**	**2.90**	**3.50**	**3.15**	**3.38**	**3.25**	**2.90**	**2.68**	**2.93**	**3.08**	**2.85**	**3.18**	**2.95**	**3.49**	**3.40**	**3.18**