Ultra Low Cost Solar Water Heater

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Engineering Analysis

Document

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1. Introduction

In the United States, a high startup cost combined with a lack of knowledge about solar water heaters has contributed to low usage. Solar water heaters are a viable way of reducing energy consumption as well as reducing a household's carbon footprint. Solar energy is abundant and free which makes it the best option for a low cost energy source. Many factors must be considered when looking to reduce cost and improve or maintain efficiency of solar water heaters.

The U.S. Environmental Protection Agency (EPA) wants to change how solar water heaters are perceived which drove them to fund the project through the P3 – People, Prosperity, and the Planet Award Program. This program gives students the opportunity to research, develop, and design solutions to real world problems involving the sustainability of society as a whole. This program was developed in order to meet the technical needs of society in relation to a sustainable future. The following document contains engineering analysis and results for different collectors to be tested in the following weeks.

The engineering analysis of the solar collectors included three different designs: Bread Box, Parabolic, and Flat Plate collector. This analysis focuses in the collectors' solar absorption per its area per its cost. Several assumptions were made to perform this analysis. Balance equation and heat transfer equations were used to obtain most of our results. Equations were conducted from the heat transfer book. We based our cost analysis of the whole system off of local hardware stores. We based our area of the systems off of the concept design of each specific collection system.

2. Bread Box Collector Analysis

The following will describe an analysis process designed to calculate the amount of heat transfer that can move into the water of a "bread box" style solar water heater. Figure 1 shows the layout of the physical design aspect of the bread box. It is important to note that the heat flux that hits the collector surface has been previously calculated through Excel spreadsheets. The q" value for a bread box with an area of $18m^2$ is 1309.83. Some temperature values were also properly assumed. Most values were chosen due to similar experiments that have been done on the materials used in our analysis.

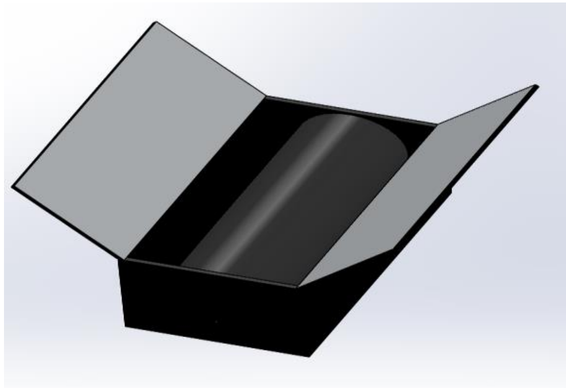

Figure 1: Bread Box Solar Water Heater

The entire analysis relies on a simple energy balance that allows you to solve for the q_{in} value. This q_{in} is the ultimate goal because it tells us how much heat transfer is making it to the water in our system. The energy balance can be seen below in. This equation stems off the fact that heat flows into the system as well as out of the system through radiation, convection, and conduction.

$$q_{solar} = q_{radiation} + q_{losses} + q_{in}$$

Where:

q_{solar} = the heat flux that hits the collector
$q_{radiation}$ = the q loss due to radiation
q_{losses} = any losses due to convection or conduction
q_{in} = heat transfer into the water

The equation that governs the losses due to radiation can be seen directly below. This equation is assuming that the medium in not inside an enclosure.

$$q_{radiation} = A_s \varepsilon \sigma (T_{s,o}^4 - T_\infty)$$

To calculate the losses due to convection and conduction, a thermal resistance network must be built. The governing equations for resistances due to convection and conduction can be seen below in the R values seen below. The resistance to convection has an h value and this value can either be assumed to be free convection of air or can be calculated. For this analysis, the h value was logically assumed and its value was made arbitrary so different scenarios could be tested.

$$R_{cond} = \frac{L}{kA}$$

$$R_{conv} = \frac{1}{hA_c}$$

$$R_{cond} = \frac{r_o/r_i}{2\pi kL}$$

Once all the resistances are known, it is possible to find the losses from ambient air to inside the bread box, and then subsequently from the inside of the box into the water. The governing equation for the losses into the box can be seen below.

$$q_{conv,air} = \frac{T_{air,i} - T_{air,o}}{R_{conv,o} + R_{cond,glass} + R_{conv,i}}$$

$$q_{cond,pipe} = \frac{T_s - T_{air,i}}{R_{pipe}}$$

By simply summing up the two above equations, we can figure out all the losses due to conduction and convection in the system. The equation below shows this relationship.

$$q_{losses} = q_{conv,air} + q_{cond,pipe}$$

The only value left to calculate is the q_{in} value. This value is governed by the q_{in} equation below.

$$q_{in} = \frac{T_{s,o} - T_{s,i}}{R_{pipe}}$$

This equation leaves us with two unknowns but if we use the energy balance, a value for $T_{s,i}$ can be easily calculated. When calculated, the $T_{s,i}$ value was 0.4K less than $T_{s,o}$. The final calculation was done to get a final value of a q_{in} = 776.7 W. This value is the amount of heat that is actually getting from the sun into the water that sits in the collector. If we know the amount of heat, a full absorption/area/cost analysis can be completed.

3. Parabolic Collector Analysis

During the brainstorming process our team originally thought that an involute collector would be much easier to construct accurately. This still holds true, however, using a 1 inch pipe we determined the collector would have a very small collection area. As a result, we decided to use a parabolic shape, despite the difficulties that may arise during construction. The following analysis determines the total heat transfer in to the water of a parabolic concentration collector. Figure 2 below shows the basic structure to the parabolic collector design.

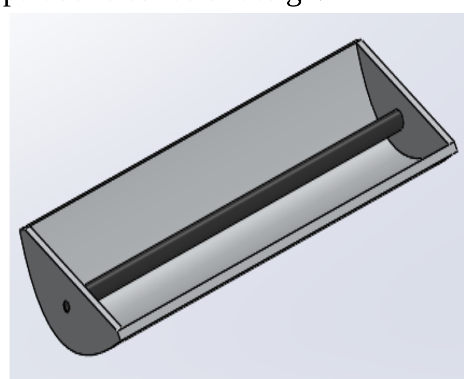

Figure 2: Parabolic Collector

Energy Balance

From the previous methodology used to determine the total heat flow q_{solar} into the system, an energy balance must be created. Heat flows out of the system through convection and radiation, and continues to the water as q_{in}.

$$q_{solar} = q_{radiation} + q_{losses} + q_{in}$$

The heat out through radiation is determined by the following equation which assumes a small object in large surroundings.

$$q_{radiation} = A_s \varepsilon \sigma \left(T_{s,o}^4 - T_\infty\right)$$

where A_s is the surface area of the object, in this case the pipe which the solar energy is concentrated. The emissivity $\varepsilon = .97$ for a flat black painted surface or $\varepsilon = .13$ for the surface of galvanized steel, is the emissivity of the pipe.. The Stefan-Boltzmann constant $\sigma = 5.67 * 10^{-8}$. Also, $T_{s,o}$ is the outside temperature of the pipe, and T_∞ is the temperature of the outside air assumed to be 20° C. q_{losses} also denoted $q_{conv,o}$ is the heat loss through convection on the outside of the pipe.

$$q_{conv,o} = \frac{(T_{s,o} - T_\infty)}{R_{conv,o}}$$

Where,

$$R_{conv,o} = \frac{1}{\bar{h}2\pi r_o L}$$

is the convective resistance on the outside of the pipe. In this equation r_o is the outside diameter of the pipe, and L is the length of the pipe. In our design $r_o = .0142$ meters for a 1 in galvanized steel pipe and $L = 5$ meters. The convection coefficient $\bar{h}$ is determined by the Nusselt Number $\overline{Nu_d}$. The equation for $\overline{Nu_D}$ depends on the Reynolds Number Re_D defined by:

$$Re_D = \frac{VD}{v}$$

where V is the velocity of the outside air assumed to be 8 mph for the average wind speed in flagstaff. $D = D_o = 2r_o$ (the outside diameter of the pipe). The kinematic viscosity $v = 1.53 * 10^{-5}$ for air at an assumed 20° C. This yield a $Re_D = 6670.85$. The Prandtl Number for air at the same temperature: $Pr_o = .709$. Based on Re_D and Pr_o, the equation for Nusselt Number is:

$$\overline{Nu_D} = \frac{\overline{h_o} D_o}{k_{air}} = C Re_D^m Pr_o^{1/3}$$

Where $k_{air} = 0.0257$ is the thermal conductivity of air at 20° C, and $C = 0.193$ and $m = 0.618$ are determined by tables based on the Reynolds number. This yields an outside convection coefficient $\overline{h_o} = 35.39$ [W/m^2-K]. Substituting that into the outside convection resistance yields $R_{conv,o} = 0.06225$.

Using r_o, $r_i = 0.0217$, $k_{steel} = 54$, and L;

$$R_{cond} = \frac{\ln(\frac{r_o}{r_i})}{2\pi k_{steel} L} = 6.68 * 10^{-5}$$

Where

$$q_{cond} = \frac{(T_{s,o} - T_{s,i})}{R_{cond}}$$

$T_{s,i}$ is the inside surface temperature of the pipe.

Assuming an inside water temperature, $T_w = 293$ K a Reynolds Number must be calculated to find the inside convection coefficient $\overline{h_i}$. The Reynolds Number $Re_D = 504.27$ indicates laminar flow on the inside of the pipe. Based on the Reynolds Number, the equation:

$$Nu_D = \frac{hD_i}{k_{water}}$$

and assuming a small change in water temperature along the pipe yields $\overline{h_i} = 103.47$.

$$R_{conv,i} = \frac{1}{\overline{h_i} 2\pi r_i L} = 0.024223$$

$$q_{conv,i} = \frac{(T_{s,i} - T_{water})}{R_{conv,i}}$$

Using a thermal circuit it can be concluded that,

$$q_{in} = \frac{(T_{s,o} - T_{water})}{R_{conv,i} + R_{cond}}$$

Substituting into the energy balance equation:

$$q_{solar} = A_s \varepsilon \sigma \left(T_{s,o}^4 - T_\infty\right) + \frac{(T_{s,o} - T_\infty)}{R_{conv,o}} + \frac{(T_{s,o} - T_{water})}{R_{conv,i} + R_{cond}}$$

And solving for $T_{s,o}$, yields $T_{s,o} = 31.853$. Substituting that value into q_{in} gives the absorption of the parabolic collector,

$$q_{in} = 737.0396 \text{ W}$$

The same steps were repeated using an unpainted galvanized pipe, which gave a slightly lower $q_{in} = 514$ W.

4. Flat Plate Collector Analysis

The flat plate collector is the most commonly used solar collector in industrial and commercial applications which was a contributing factor to why it was chosen for analysis. A simple structure of the flat plate collector can be seen below in Figure 3. The following analysis determines the total heat transfer in to the water of a flat plate collector made from common materials.

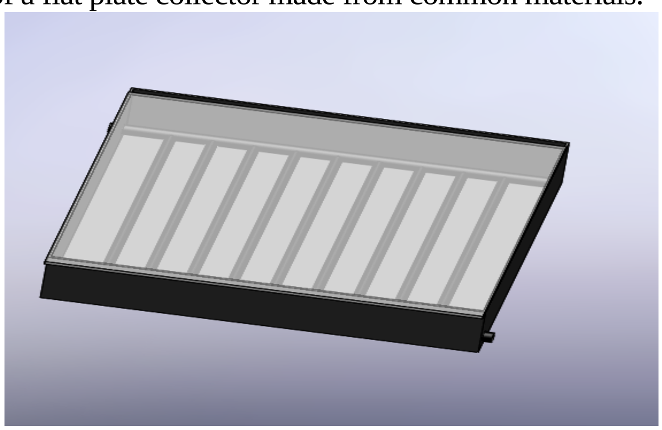

Figure 3: Flat Plate Collector

Energy Balance

The goal of the analysis is to determine the amount of heat flux transferred into the water. To start this calculation the total amount of absorbed radiation must be calculated using the equation below. Table 1 provides typical material properties for reflectivity, absorptivity, and transmissivity.

$$q_{solar} = q_{radiation} A_s \rho \alpha \tau$$

Where:

q_{solar} = Solar irradiance at pipe surface
$q_{radiation}$ = Solar irradiance
ρ = Reflectivity
α = Absorptivity
τ = Transmissivity

Table 1: Material Properties

Material	Reflectivity	Absorptivity	Transmissivity
PVC		0.94	
Galvanized Metal		0.65	0.94
Mylar			0.95
Glass	0.85		
Plexiglass	0.75		
Plastic Sheeting	0.65		
Flat Black Paint		0.97	
Lava Rock		0.75	

The q_{solar} can then be used to calculate the total useful heat flux based on the equation below.

$$q_{solar} = q_{radiation} + q_{losses} + q_{in}$$

Where:

$q_{radiation}$ = Radiation heat loss
q_{losses} = Losses from convection and conduction
q_{in} = Heat flux into the water

The heat out through radiation is determined by the following equation which assumes a small object in large surroundings.

$$q_{radiation} = A_s \varepsilon \sigma \left(T_{s,o}^4 - T_\infty\right)$$

Where:

$\varepsilon =$ Emissivity of surface
σ= Stefan-Boltzman constant
T_s= Surface temperature
T_∞= Ambient temperature of surroundings

q_{losses} can be calculated, which is the change in temperature divided by the resistance of the medium through which it must travel.

$$q_{losses} = \frac{(T_s - T_\infty)}{R_{conv,air} + R_{cond,glass} + R_{cond,air}}$$

Where:

$$R_{convenction,a} = \frac{1}{h_{air} A_s} = \frac{K}{L}$$

$$R_{conduction,g} = \frac{L}{K_{glass} A_s}$$

$$R_{conduction,a} = \frac{L}{K_{air} A_s}$$

The convection resistance is calculated using the Reynolds number for forced convection assuming there is wind blowing over the outside surface of the glass. The Reynolds number is calculated using the equation below. The convection off the glass accounts for the largest heat loss from the system.

$$Re_D = \frac{VL}{v}$$

Where:

V= Velocity of the wind
L= Length to the from the farthest edge
ν= viscosity of the air

The convective resistance of the air can be considered as conduction due to the fact that the Rayleigh number was calculated to be 671 which is less than 1708. The Rayleigh number compares the ratio of buoyancy forces to viscous forces. By definition of an enclosure, the viscous forces overcome the buoyancy forces and do not allow air to circulate. This allows the convection resistance inside the enclosure to be calculated using the method of conduction.

$$Ra_L = \frac{g\beta(T_1 - T_2)L^3}{\alpha\nu}$$

Where:

g= Gravity
β= Expansion coefficient
T_1= Hot surface
T_2= Cold surface
L= Distance from glass to pipe
α= Thermal diffusivity

The basic equation for conduction can be used to calculate the conductive resistance of the glass based on the thickness and thermal conductivity of glass.

Summing the losses and subtracting them from the useable solar heat flux gives the heat flux q_{in}'' into the water. Assuming the water from the city comes in at roughly 283 degrees kelvin, the temperature of the water after passing through the solar collector can be obtained.

$$T_{mo} = T_{mi} + \frac{q_{in}PL_{pipe}}{\dot{m}C_p}$$

Where:

T_{mo}= Fluid temperature at outlet
T_{mi}= Fluid temperature at inlet
P= Perimeter
L_{pipe}= Length of pipe
$\dot{m}$= Mass flow rate of fluid
C_p= Specific heat

Table 2 provides data on inlet temperature of city supplied water in cities across the nation. Calculated values for different collector styles can be seen in Table 3 below. The data displayed in Table 3 is given assuming an average inlet temperate of 283°K.

Table 2: Water Inlet Temperatures

Location	Average Inlet Temperature (°F)	Location	Average Inlet Temperature (°F)
Phoenix, AZ	82.3	Denver, CO	61.3
Detroit, MI	49.9	Boston, MA	59.3
St. Louis, MO	61.3	Milwaukee, WI	46.0
Dallas, TX	68.3	New Orleans, LA	64.9

Table 3: Heat Flux Values for Different Materials

		q''_{in} $(\frac{W}{m^2})$	T_{mo} (°K)
Materials 1	PVC Glass	714.776	284.628
Materials 2	Galvanized Pipe Glass	714.784	284.615
Materials 3	PVC Plastic	558.235	284.272
Materials 4	Galvanized Pipe Plastic	558.243	284.262

From the results in Table 3 it is determined that there is very little difference in performance between the different configurations. Material selection will be determined by the cost of the material in order to optimize the absorption per cost per area metric.

5. Gantt Chart

The Gantt chart shows the timeline for the project. Milestones in chart are indicated by red diamonds. The green sections relate to the project description and analysis, where the blue sections are related to preparing and researching for project tasks. Last milestone we met was concept generation and selection presentation. However, after analyzing the collectors, we updated our Gantt chart by adding a new section for circulation analysis. The circulation analysis will take place after the collector's analysis presentation which is shown as a milestone. The rest of tasks timeline did not change. Our last milestone for this semester will be submitting the design proposal and presentation in 3rd of December.

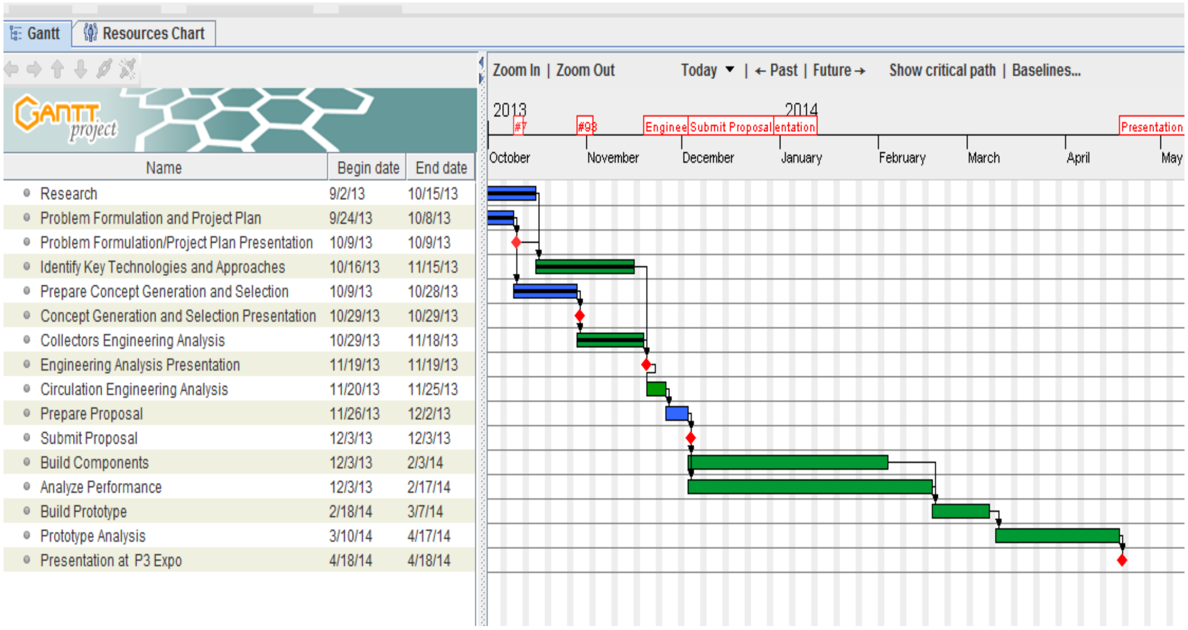

Figure 4: Gantt Chart

6. Conclusion

<u>Area/$</u>

Bread box collector and circulation system:

- With an area of 4.6 m^2 and a cost of \$201.82 the area per cost of the bread box collector with plastic covering is .02256, giving it the highest area per cost value out of all of the other collector designs.
- With an area of 4.6 m^2 and a cost of \$279.80 the area per cost of the bread box collector with glass covering is .01627.

Parabolic collector and circulation system:

- Using an area of 1.16 m^2 and a cost of \$255.64 the area per cost of the parabolic collector with galvanized, unpainted piping is .01194.
- Using an area of 1.16 m^2 and a cost of \$260.23 the area per cost of the parabolic collector with galvanized, black painted piping is .01140.

Flat plate collector and circulation system:

- With an area of .93 m^2 and a cost of \$488.42 the area per cost of the flat plate collector with galvanized pipe with no spacing in between each pipe is .00282, giving is the lowest area per cost value out of all the other collector designs.
- With an area of .93 m^2 and a cost of \$234.9 the area per cost of the flat plate collector with a rock bed as a thermal reservoir is .01214.

Area/$ conclusion: It is obvious from this analysis that the bread box collector and circulation system utilizing a plastic cover is the most affordable per its area.

<u>Absorption/area/$</u>

Bread box collector and circulation system:

- With an absorption of 654.32 W for plastic covering, an area of 1.67 m^2, and a cost of $181.31 the absorption per area per cost is 2.16.
- With an absorption of 776.6 W for glass covering, an area of 1.67 m^2, and a cost of $201.36 the absorption per area per cost is 2.31.

Parabolic collector and circulation system:

- Using an absorption of 514.07 W for galvanized, unpainted piping, an area of 1.16 m^2, and a cost of $255.64 the absorption per area per cost is 1.73.
- Using an absorption of 737.04 W for galvanized, black painted piping, an area of 1.16 m^2, and a cost of $260.23 the absorption per area per cost is 2.44.

Flat plate collector and circulation system:

- Using an absorption of 738.48 W for galvanized pipes with no spacing, an area of .93 m^2, and a cost of $488.41 the absorption per area per cost is 1.63.

Absorption/area/$ conclusion: It is obvious from this analysis that the parabolic collector and circulation system utilizing galvanized, black painted pipe is the most affordable per its absorption per area and will be the design that team 13 will use as a final concept.

7. References

- Energy Protection Agency (EPA). (2013). *P3:People, Prosperity, and the Planet Student Design Competition for Sustainability* . Washington, DC: EPA. http://www.epa.gov/P3/. Accessed October 2013

- U.S. Department of Energy (DOE). (2012). *Building Energy Codes Program*. Washington, DC:

- U.S. Department of Energy. http://www.energycodes.gov/adoption/states/. Accessed October 2013.
- Raisul Islam, K. Sumathy and S. Ullah Khan, "Solar water heating systems and their market trends," *Renewable and Sustainable Energy Reviews*, vol. 17, pp. 1-25, 1, 2013.
- U.S. Department of Energy (DOE). (2005). *Residential Energy Consumption Survey 2005*. Washington, DC: Energy Information Administration. http://www.eia.doe.gov/emeu/recs/. Accessed October 2013.
- Energy Information Administration (EIA). (2005). *Office of Energy Markets and End Use, Forms EIA-457 A-G of the 2005 Residential Energy Consumption Survey.* Washington, DC: EIA.
- American Society of Heating and Air-Conditioning Engineers (ASHRAE). (1987). *Methods of Testing TO DETERMINE THE SOLAR PERFORMANCE OF SOLAR DOMESTIC WATER HEATING SYSTEMS*. Atlanta, GA: ASHREA Standard- 95.

- American Society of Heating and Air-Conditioning Engineers (ASHRAE). (1987). *Methods of Testing to Determine the Performance of Solar Collectors*. Atlanta, GA: ASHREA Standard-93.
- M. Raisul Islam, K. Sumathy, Samee Ullah Kahn. (2012). *Solar water heating systems and their market trends*. Atlanta, GA: Renewable and Sustainable Energy Reviews. http://www.elsevier.com/locate/rser/. Accessed October 2013.
- Database of State Incentives for Renewables and Efficiency (DSIRE). (2012). *Arizona Incentives/Policies for Renewables and Efficiency*. Raleigh, NC: North Carolina State University.
- http://www.dsireusa.org/incentives/index.cfm?re=0&ee=0&spv=0&st=0&srp=1&state=AZ/. Accessed October 2013.
- U.S. Department of Energy (DOE). (2012). *Building Codes and Regulations for Solar Water Heaters*. Washington, DC: U.S. Department of Energy. http://energy.gov/energysaver/articles/building-codes-and-regulations-solar-water-heating-systems/. Accessed October 2013.
- U.S. Department of Energy (DOE). (2012). *Building Energy Codes Program*. Washington, DC: U.S. Department of Energy. http://www.energycodes.gov/adoption/states/. Accessed October 2013.
- Energy Protection Agency (EPA). (2013). *P3:People, Prosperity, and the Planet Student Design Competition for Sustainability* . Washington, DC: EPA. http://www.epa.gov/P3/. Accessed October 2013