

SAE Mini Baja Drivetrain

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Team 02

Engineering Analysis

REPORT

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I. Introduction

This report will go into further depths of the actual engineering analysis and formulation of the SAE Baja Team's drivetrain. The contents describe the formal equations that describe our concept generation as well as the layouts and explanations of the final system the group is preparing. Below we have generated a gear ratio from our given constraints, reasonable assumptions and our goal speed and torque. We analyzed two different concepts, an automatic transmission and a continuous variable transmission. The analysis of ratios and torque revealed substantial evidence that the continuous variable transmission far exceed the capability of the automatic gear box that we had intended to use.

II. Goal

a. Torque

In the hill climb event, the Baja vehicle will be expected climb an incline of significant difficulty. The team assumed the incline to be approximately 30 degrees. Through the inspection of previous courses, as a group we felt this would be the maximum angle in any hill climb we might encounter. In order to complete the incline, the force on two wheels will need to be greater than the component force of gravity along the incline, which is G1 in the figure below:

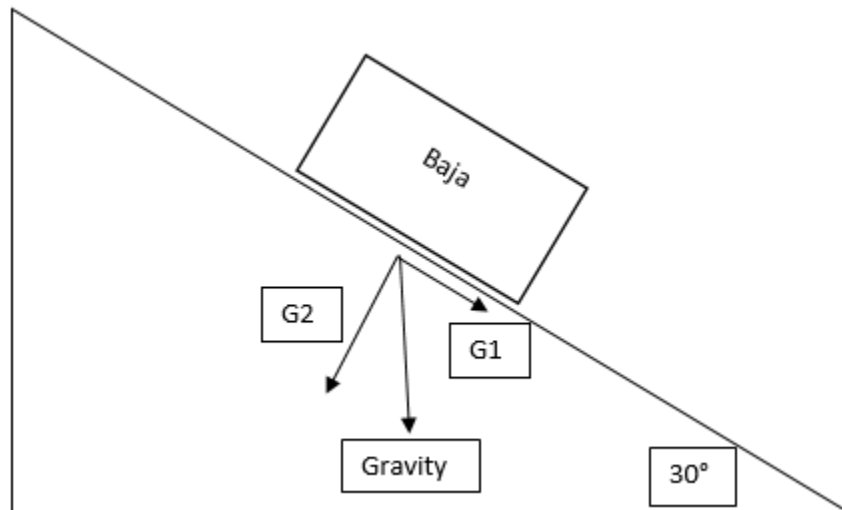


Figure 1: Free Body Diagram of Baja for Hill Climb Event

- $G_1 = G * \sin \theta = 600\text{lb} * \sin 30 = 300 \text{ lb}$
- Force per wheel = 150 lb
- Torque per wheel = $150\text{lb} * \frac{D}{2} = 150\text{lb} * 11.5 \text{ in}/12 = 143.75 \text{ lb-ft}$
- Total torque (T_t) = $287.5 \text{ lb} - \text{ft}$

From the equations above we can assume that the minimum torque that needs to be transferred to the wheels is 290 lb-ft.

b. Speed

Table 1: Tennessee 2013 Acceleration Event

Rank	Car No	School	Team	Time Run 1	Time Run 2	Best Time	Acceleration Score (75)
1	1	Cornell Univ	Big Red Racing	3.870	3.861	3.861	75.00
2	52	Michigan Tech Univ	Blizzard Baja	3.950	3.872	3.872	74.70
3	6	Univ of Maryland - Baltimore County	UMBC Racing	3.902	3.957	3.902	73.86
4	78	Univ of Maryland - College Park	Terps Racing	3.906	3.974	3.906	73.75
5	73	LeTourneau Univ	Renegade Racing	3.935	3.916	3.916	73.48
6	3	Rochester Institute of Technology	RIOT Racing	3.999	3.924	3.924	73.26
7	44	Ohio Northern Univ	Polar Bear Racing	3.945	3.955	3.945	72.67
8	36	Universite de Sherbrooke	Sherbrooke Racing Team	4.011	3.992	3.992	71.37
9	57	Univ of Wisconsin - Madison	UW Baja	4.129	4.037	4.037	70.13
10	45	Univ of Arkansas - Fayetteville	Racing Razorbacks	4.043		4.043	69.96

(Source: sae.org)

From the table we can see the top team have an average time of 4 seconds to finish a 100 foot course. Assuming that the Baja keeps accelerating with the average acceleration during that time. We can calculate the maximum velocity

$$\text{Distance} = \text{Max Velocity} * \text{time} / 2$$

$$\text{Max velocity} = \text{Distance} * 2 / \text{time} = 100 \text{ ft} * 2 / 4\text{s} = 50 \text{ ft/s} = 34 \text{ mph}$$

Based on the result, 40 mph is the goal for max speed that the team has set out to obtain.

III. Analysis for CVT system

The analysis of the continuously variable transmission essential provide the gear ratios that would be required to obtain the goals introduced previously. Through the analysis the team was forced to make specific assumption and decisions on criteria such as wheel diameter and total weight. Unfortunately we were unable to acquire such information as a total weight of the vehicle or an exact degree for the slope of the Hill Climb event. Thus, these became assumptions along with the frontal area of the vehicle. These are all clearly laid out below.

a. Assumption and Variables

- Wheel diameter(D): 23 inch
- Total weight (W): 600 lb (including the driver)
- Slope of the hill (θ): 30 degree
- Reduction ratio (r_r): 12:1
- Efficiency of CVT(N_{cvt}): 88%
- CVT: high ratio (r_{h-cvt}) : 0.5 low ratio (r_{l-cvt}) : 3
- Start RPM for CVT is 800 rpm and high speed ratio occur at 3600 rpm, ratio varies linearly within 800 rpm and 3600 rpm. We find the following relationship:

$$r_{cvt} = \begin{cases} 0 & \text{for } rpm < 800 \\ 3 - \frac{2.5 * (rpm - 800)}{2800} & \text{for } 800 < rpm < 3600 \\ 0.5 & \text{for } 3600 < rpm \end{cases}$$

- Total ratio: high ratio (r_h), low ratio (r_l)

b. Continuously Variable Transmission Set-Up

The CVT has initial gear ratios of .45:1 (high) and 3.1:1 (low). This however was not was not ideal for the goals that have been established. Thus, the group had to consider a secondary reduction or in this case two. As stated above in the assumptions and variables, the total reduction ration should be a total of 12:1. For the volume provided to us by the frame team, which is approximately 6.3 cubic feet, our team put together this simple lay out of the reduction system as seen in Figure 2. In Figure 3 we depict how the engine, CVT and reduction system might sit with in the frame. As you can see because of the odd shape of the rear, to optimize the space, the engine should be mounted

approximately 17 inches above the bottom of the frame. This can be visualized in Figure 4. This will allow for ample space to implement the reduction system and eventually our braking system.

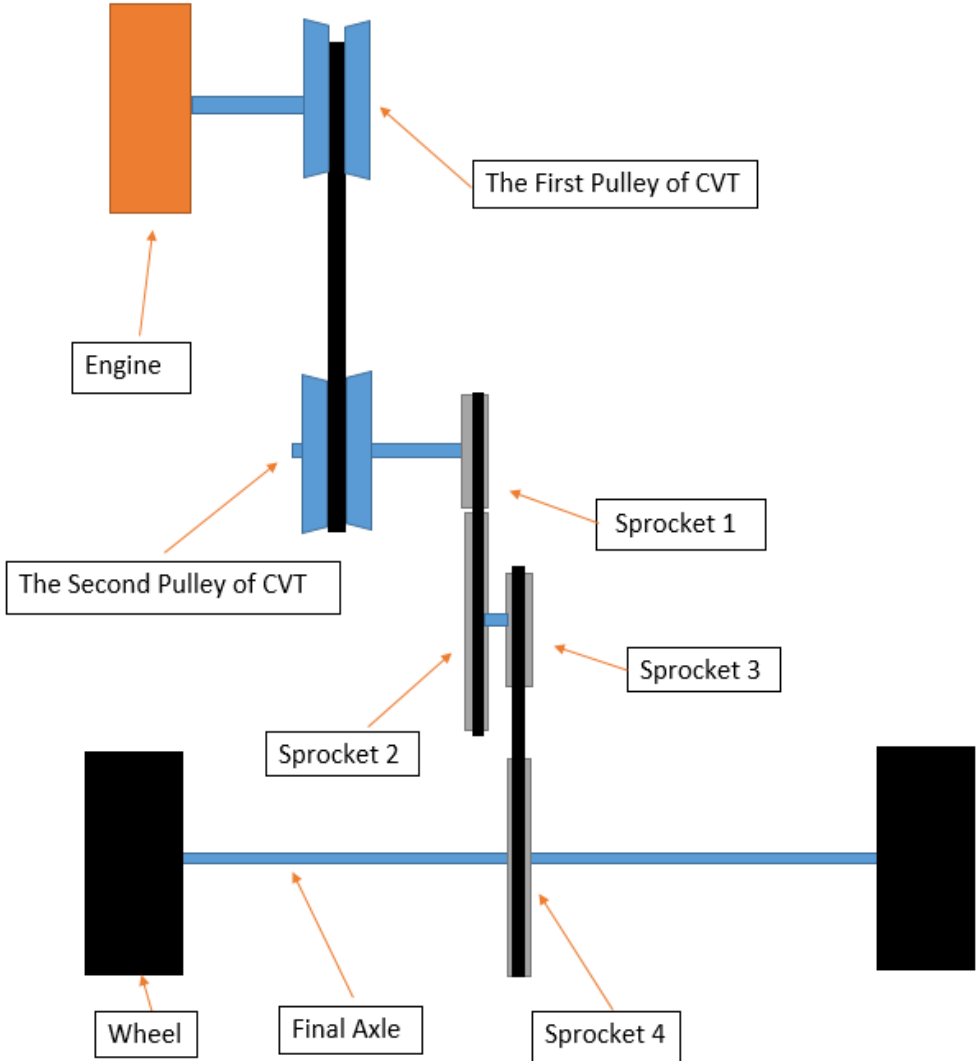


Figure 2: Basic Concept of CVT Drivetrain System

The reduction contains 4 sprockets with different teeth

$n1=16$	$n3=16$
$n2=64$	$n4=48$

Sprocket 1&2 is the first stage with 4:1 ratio, sprocket 3&4 is the second stage with 3:1 ratio.

The total reduction ratio is 12:1.

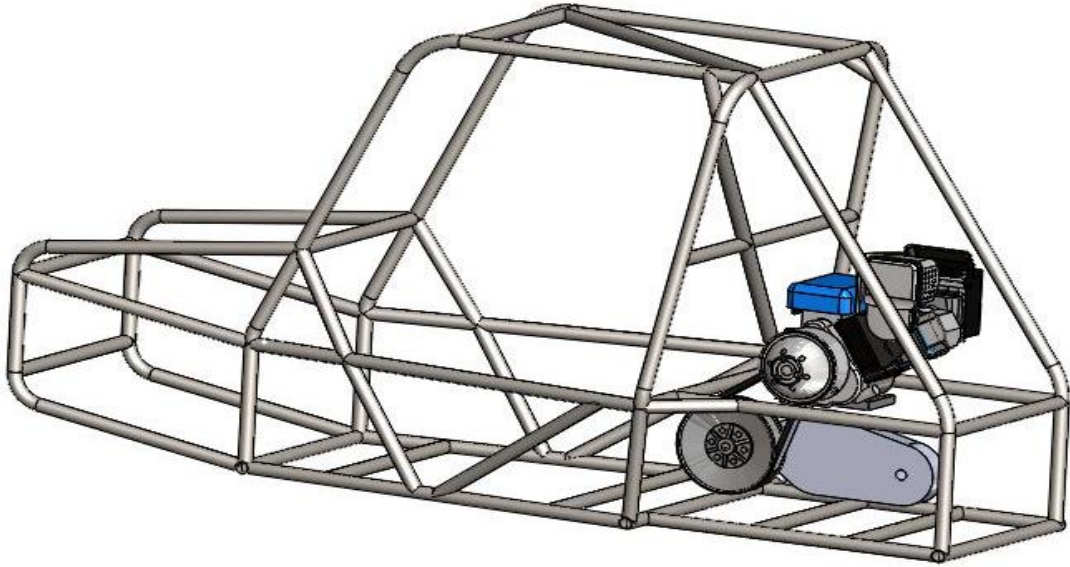


Figure 3: 3-D Drawing of CVT Drivetrain System

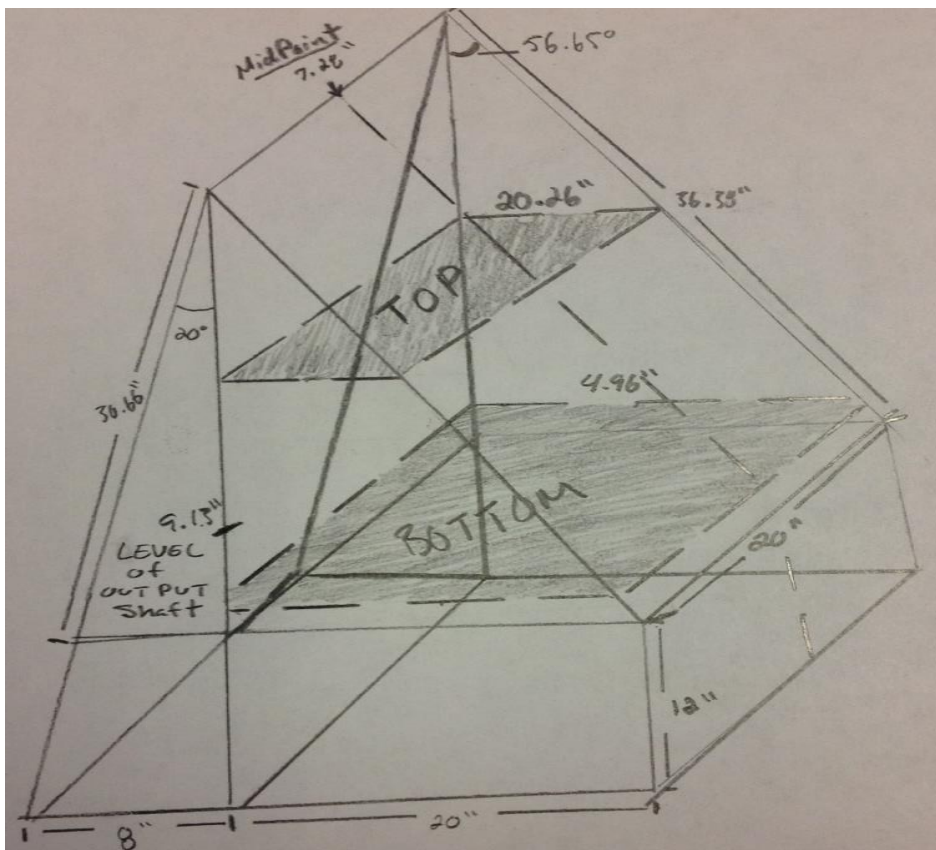


Figure 4: Simple depiction of the lay out of the rear of the frame and the prospected optimal placement of the engine

c. Calculations

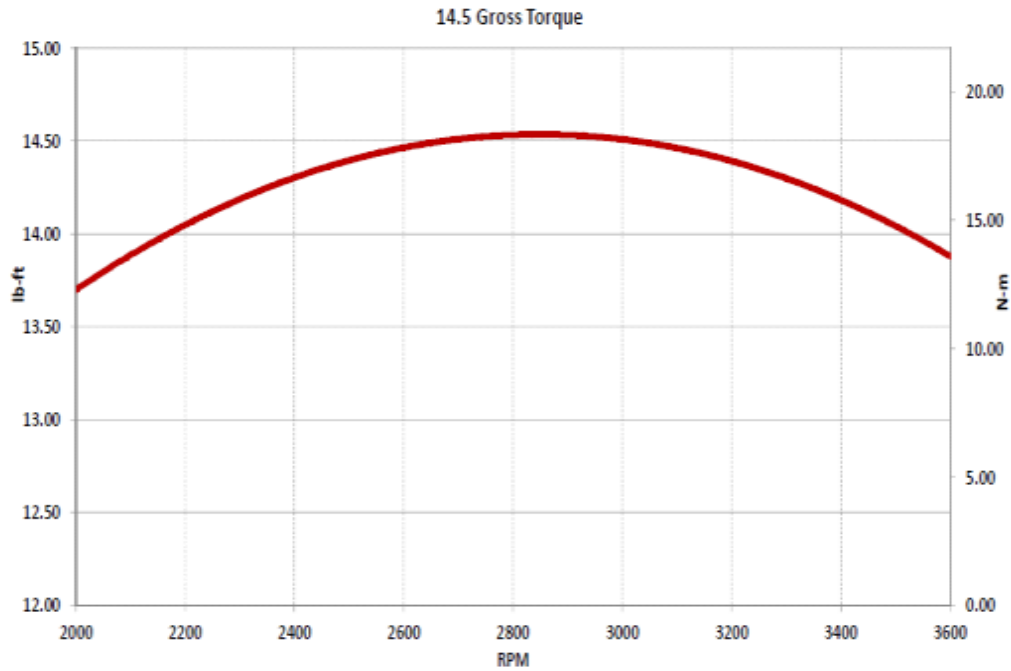


Figure 5: Motor Torque Curve. (Source: Briggs & Stratton)

From the graph above we obtain the RPM and torque output from the engine. Then we calculated the following with our assumptions:

- CVT ratio = $3 - \frac{2.5*(rpm-800)}{2800}$ for $800 < rpm < 3600$
- Total ratio = $r_{cvt} * r_r * N_{cvt} = r_{cvt} * 12 * 0.88$
- Torque on the wheel = Torque output * Total ratio * N_{cvt}
- Speed = $\frac{D * RPM * \pi}{total\ ratio * 12 * 60} * 0.68 = \frac{23\ in * RPM * \pi}{total\ ratio * 12 * 60} * 0.68$

With the equation above we made the table below.

Table 2: This table displays our numerical data as it relates to our assumptions and the equations

Engine rpm	Torque output (lb-ft)	CVT ratio	Total ratio	Torque on wheel (lb-ft)	Speed (mph)
1800	13.20	2.107	22.251	293.719	5.52
2000	13.70	1.929	20.366	279.010	6.70
2200	14.10	1.750	18.480	260.568	8.12
2400	14.30	1.571	16.594	237.298	9.87
2600	14.45	1.393	14.709	212.539	12.06
2800	14.52	1.214	12.823	186.188	14.90
3000	14.50	1.036	10.937	158.589	18.72
3200	14.40	0.857	9.051	130.341	24.13
3400	14.20	0.679	7.166	101.753	32.38
3600	13.80	0.500	5.280	72.864	46.53

The max torque is 293.7 lb-ft and the max speed is 46.53 mph which satisfy the team’s intended goals. Thus, our assumption for the CVT ratio is realistic and obtainable. Based on the 0.5 high ratio and the 3 low ratio, the team chose the CVT: *PULLEY SERIES 0600 AND DRIVEN PULLEY SERIES 5600* from CVTech-AAB Inc. This CVT provides a range of 0.45 high ratio to 3.1 low ratio that will be compatible with the design. However, it changes the equation for CVT ratio slightly. Thus:

$$\text{CVT ratio} = 3.1 - \frac{2.65*(rpm-800)}{2800} \quad \text{for } 800 < rpm < 3600$$

With the new ratio we can adjust our data to provide the updated Table 2.

Table 3: This table displays our numerical data as it relates to our assumptions and the equations

Engine rpm	Torque output (lb-ft)	CVT ratio	Total ratio	Torque on wheel (lb-ft)	Speed (mph)
1800	13.20	2.154	22.742	300.191	5.40
2000	13.70	1.964	20.743	284.177	6.58
2200	14.10	1.775	18.744	264.290	8.01
2400	14.30	1.586	16.745	239.456	9.78
2600	14.45	1.396	14.746	213.084	12.03
2800	14.52	1.207	12.747	185.093	14.99
3000	14.50	1.018	10.749	155.854	19.05
3200	14.40	0.829	8.750	125.996	24.96
3400	14.20	0.639	6.751	95.862	34.37
3600	13.80	0.450	4.752	65.578	51.70

The maximum torque applied on the sprockets are followed by the equations below where:

(T is the torque output from engine, T_1 is the torque applied on the first sprocket)

$$T_1 = T * r_{cvt} * N_{cvt} = 13.20 \text{ lb-ft} * 2.154 * 0.88 = 25.02 \text{ lb-ft}$$

$$T_2 = T_1 * \frac{n_2}{n_1} = 25.02 \text{ lb-ft} * 4 = 100.08 \text{ lb-ft}$$

$$T_3 = T_2 = 100.08 \text{ lb-ft}$$

$$T_4 = T_3 * \frac{n_3}{n_2} = 100.08 \text{ lb-ft} * 3 = 300.19 \text{ lb-ft}$$

IV. Analysis for Auto System

a. Assumption and variables

- Wheel diameter(D): 23 inch
- Total weight (W): 600 lb (including a driver)
- Transmission gear ratios:
high speed gear ratio $r_{\text{auto high}}$:2.88:1, low speed gear ratio $r_{\text{auto low}}$:7.49:1
- Slope of the hill(θ): 30 degree
- Efficiency of automatic transmission(η_{auto}): 85%
- Efficiency of sprockets system($\eta_{\text{secondary}}$) = 95%
- T : torque from engine
- T_m : maximum torque on the wheels

b. Automatic Transmission Set-Up

The automatic transmission provides initial gear ratios of 2.88:1 (high) and 7.49:1 (low).

Similarly to the CVT, this concept requires the implementation of a secondary reduction system.

Below in Figure 6 you can see a simple representation of the desired lay out. Though the automatic gear box takes up X space, the cubic footage provided to us by the Frame Team should still suffice. Secondly, this concept removes the necessity for a second reduction ratio. This would allow for more area when implementing our brake system as well as less area to cover when implementing chain guards as a part of the SAE safety requirements.

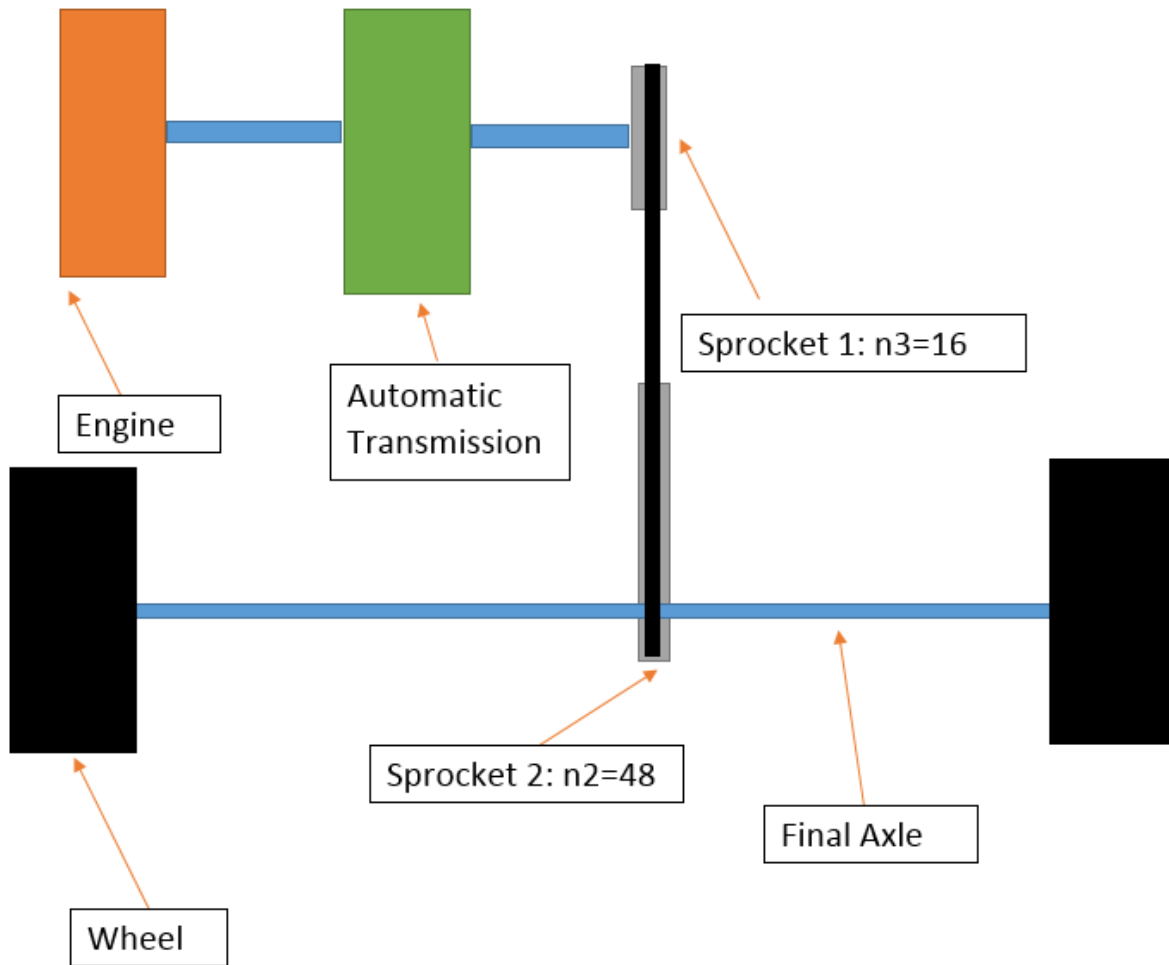


Figure 6: This diagram depicts the prospected lay out of the Automatic gear box

In the dawning of automatic transmission drive-train system, we have two sprockets

$n_1=16$: teeth of the first gear

$n_2=16$: teeth of the second gear

$n_3=16$: teeth of the third gear

$n_4=48$: teeth of the fourth gear

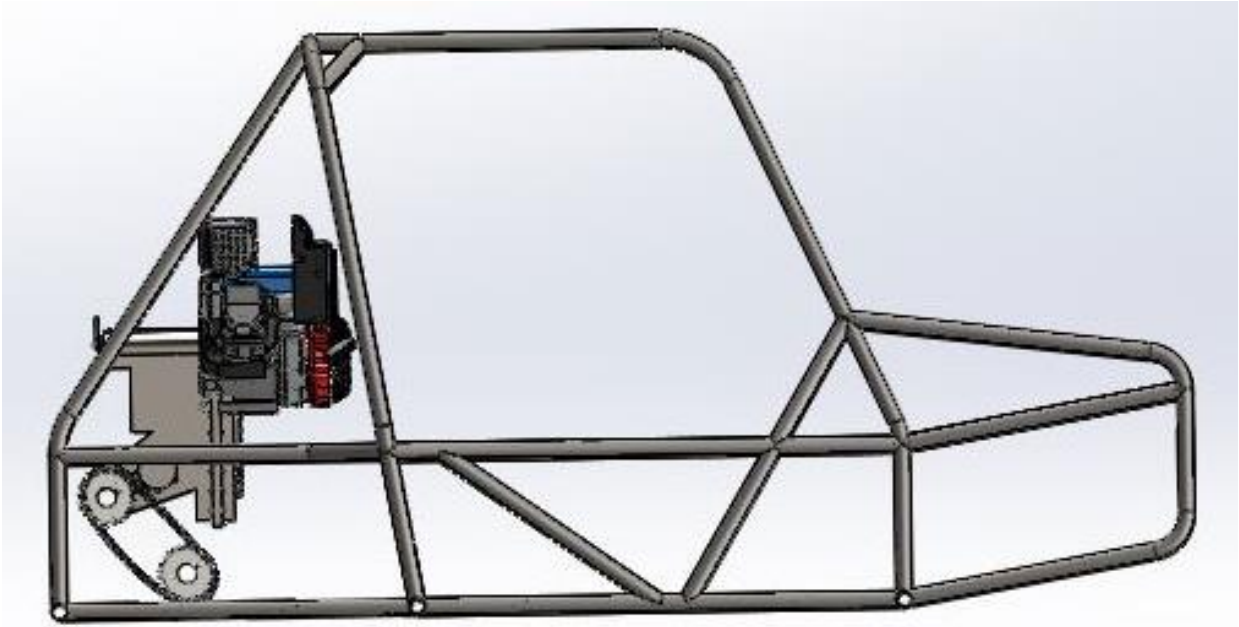


Figure 7: 3-D Drawing of Automatic Transmission Drivetrain System

c. Calculation

- Based on the teeth of each sprocket, we had our gear ratio outside of automatic transmission r_{second} ,

$$r_{second} = \frac{n_2 n_4}{n_1 n_3} = \frac{48 * 16}{16 * 16} = 3$$

- We can have the high speed ratio of the whole automatic transmission system r_h

$$r_h = r_{auto\ high} * r_{second} = 2.88 * 3 = 8.64$$

- We have low speed ratio of the whole automatic transmission system r_l

$$r_l = r_{auto\ low} * r_{second} = 7.49 * 3 = 22.47$$

- We assume our efficiency of automatic transmission $\eta_{auto} = 85\%$, the efficiency of the sprockets $\eta_{secondary} = 95\%$, T is the maximum torque we can get from the engine. We calculated the r_l at last step above. And the torque on the wheels we are going to have by using automatic transmission is 263.10 lb-ft by using the equation below

$$T_m = T * r_l * \eta_{auto} * \eta_{secondary} = 14.5\ lb\text{-ft} * r_l * \eta_{auto} * \eta_{secondary} = 263.10\ lb\text{-ft}$$

- We used the max speed V_m equation found out the max speed on the wheels

$$(V_m) = \left(\frac{23 \text{ in} \cdot \pi \cdot 3800 \text{ rpm}}{r_h \cdot 12 \cdot 60} \right) \cdot 0.68 = 30.01 \text{ mph}$$

- The maximum torque applied on the sprockets at high and low speed are:
(T is the torque output from engine, T1 is the torque applied on the first sprocket)

$$T1 = T \cdot r_{\text{auto high}} = 14.50 \text{ lb-ft} \cdot 2.88 = 41.76 \text{ lb-ft}$$

$$T2 = T1 \cdot \frac{n2}{n1} = 41.76 \text{ lb-ft} \cdot 3 = 125.28 \text{ lb-ft}$$

$$T1 = T \cdot r_{\text{auto low}} = 14.50 \text{ lb-ft} \cdot 7.49 = 108.605 \text{ lb-ft}$$

$$T2 = T1 \cdot \frac{n2}{n1} = 108.605 \text{ lb-ft} \cdot 3 = 325.815 \text{ lb-ft}$$

Based on our calculation, the maximum torque may be applied on the sprockets is 325.815 lb-ft.

V. Project Plan

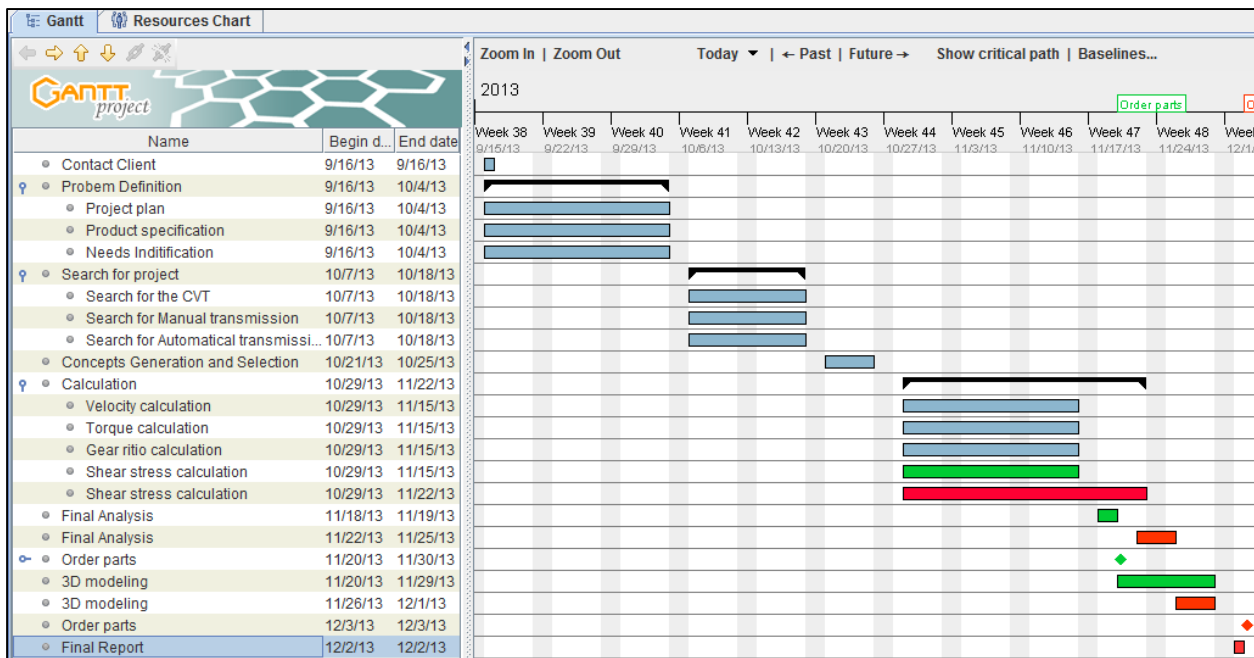


Figure 8: Updated Gantt Chart

Based on the Gantt chart in figure (8), our team is up to date on the analysis except for the shear stress analysis. This delay caused us to push our plan and compress it so that we can still finish on time. We specified fewer days to finish the analysis but our team intend to spend more time daily to meet the project deadlines. In the Gantt chart, the green bars represent the old plan

and the red bars are our updated plan which is still scheduled to finish by the deadline specified for our project.

VI. Conclusion

Our group suggested three concepts design which are automatic, manual, and CVT transmission design. Top two choice were chosen to do further analysis. To do our analysis, our team generally analyzed the overall system to find what desired torque and speed which turned to be 290 lb-ft and 40 mph respectively. As for the CVT system, it has 300.19 lb-ft torque and 51.70 mph which exceeds our expectations and should meet our goals considering friction force and power lost. So, the ratios used to calculate for the CVT were compared to existing CVT transmissions on the market and chose *PULLEY SERIES 0600 AND DRIVEN PULLEY SERIES 5600* from CVTech-AAB Inc. Then, the automatic and CVT transmissions were analyzed to find the resulted torque and speed to the wheels using each system. Based on our results, the automatic transmission has only 263.10 lb-ft for torque and 30.01 mph for speed which fails to meet our desired goals. Therefore, our team decided to complete the full analysis for the CVT transmission only as well as add the clutch so that our system will have a reverse option.

VII. References

a. CVTech-AAB

Available: http://www.numeriquetechnologies.com/cvtech/CatalogueCVTech-AAB_US_%202013.pdf

b. Seamless AMT offers efficient alternative to CVT

Available:

<http://www.zeroshift.com/pdf/Seamless%20AMT%20Offers%20Efficient%20Alternative%20To%20CVT.pdf>

c. Baja SAE Result

Available: <http://students.sae.org/competitions/bajasae/results/>

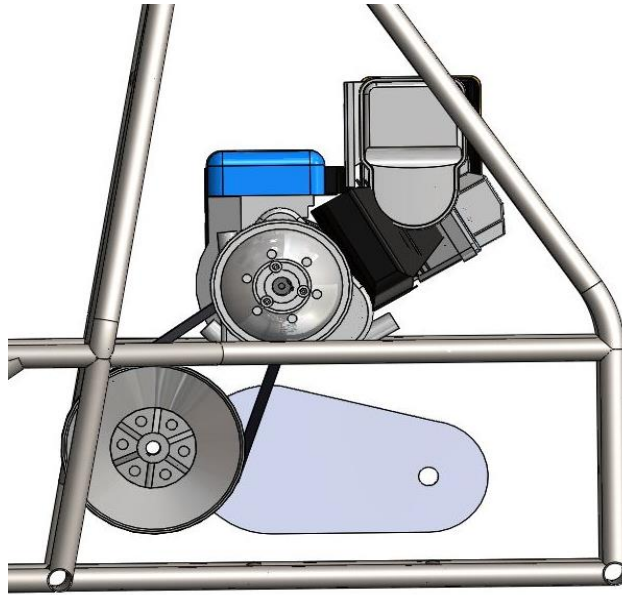
d. Kluger, M and Long, D. “*An Overview of Current Automatic, Manual and Continuously Variable Transmission Efficiencies and Their Projected Future Improvements*”. SAE 1999-01-1259.

e. Richard Budynas, and J Keith Nisbett. *Mechanical Engineering Design*. 9th. 1021. New York: McGraw-Hill, 2011. Print.

f. Marcelo de Jeus R, da nobrega, Souza Xavier Leydervan de, et al. "Modeling and Simulation of the Transmission System-Dynamic of a System equipped with a CVT for Mini-Baja vehicle." *SAE Technical paper series*. Sao Paulo: SAE Brasil, 2004. 5. Print.

VIII. Appendix

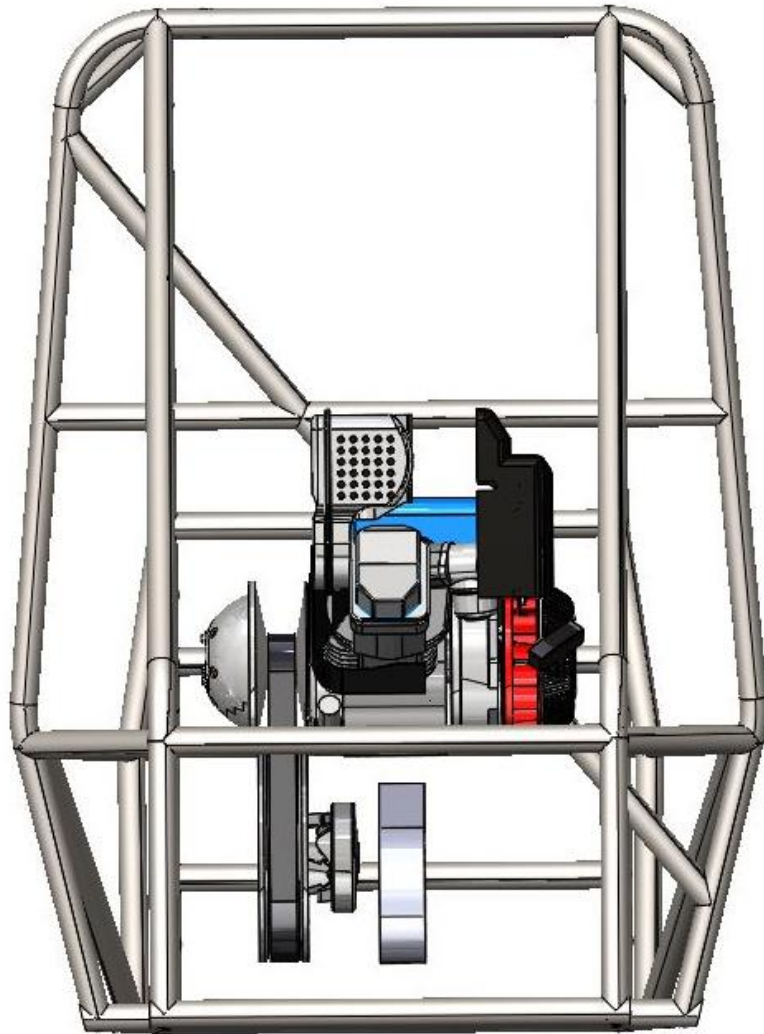
a. This Appendix provides visual aids for the CVT



i.



ii.



iii.

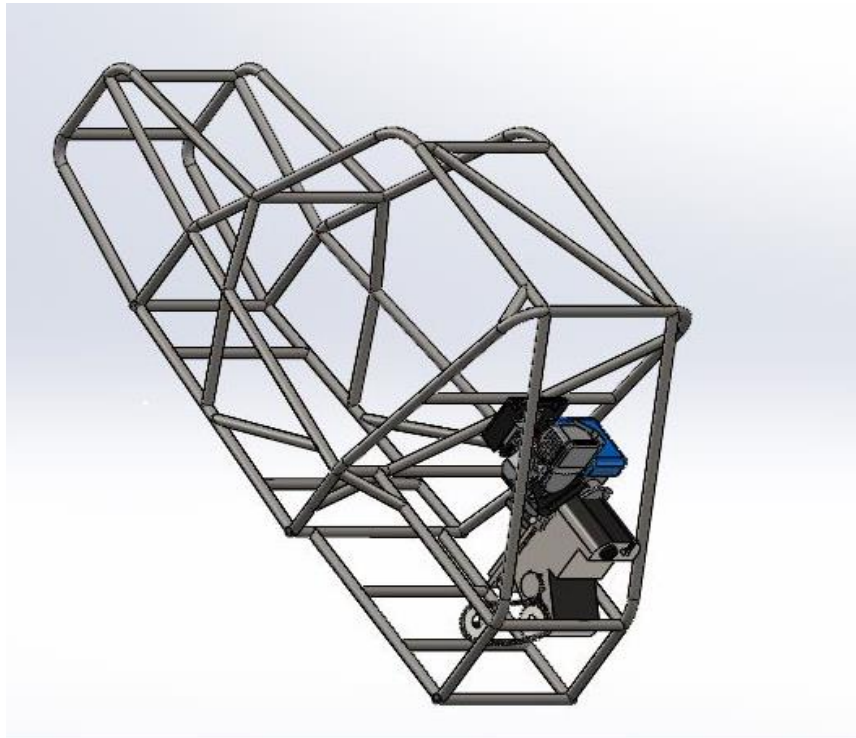
b. This Appendix Provide visual aids for the Automatic Transmission



i.



ii.



iii.